

5th World Congress on the SQUARE OF OPPOSITION



November 11-15, 2016, Easter Island



Handbook of Abstracts

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1. Fifth World Congress on the Square of Opposition

1.1. The Square: A Central Object for Thought

The square of opposition is a very famous theme coming from Aristotelian logic dealing with the notions of opposition, negation, quantification and proposition. It has been continuously studied by people interested in logic, philosophy and Aristotle during two thousand years. Even Frege, one of the main founders of modern mathematical logic, used it.

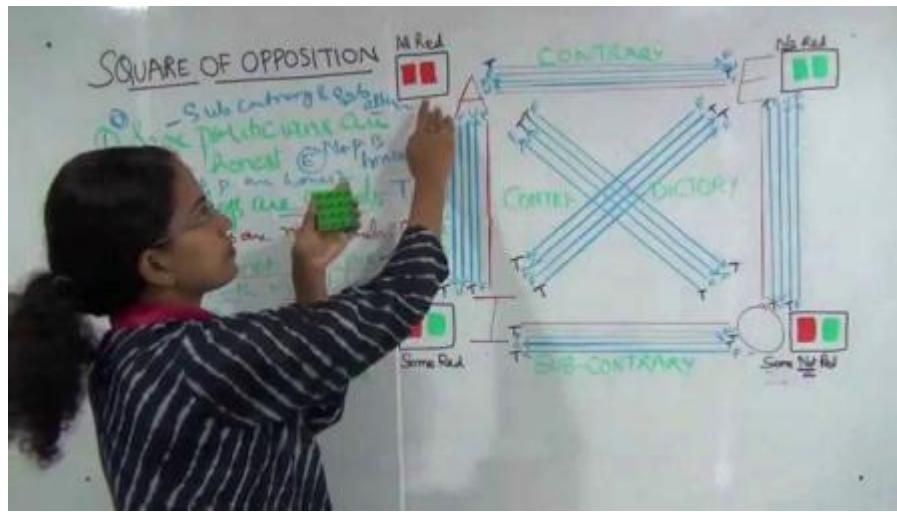


The Hexagon of Opposition of Robert Blanché was a major step in the development of the theory of opposition.

Some people have proposed to replace the square by a triangle, on the other hand the square has been generalized into more complex geometrical objects: hexagons, octagons and even polyhedra and multi-dimensional objects.

1.2. Aim of the Congress

This will be the 5th world congress organized about the square of opposition after very successful previous editions in Montreux, Switzerland, in 2007; Corté, Corsica, in 2010; Beirut, Lebanon, in 2012; and Vatican, in 2014. This is an interdisciplinary event gathering logicians, philosophers, mathematicians, semioticians, theologians, cognitivists, artists and computer scientists.



The Square of Opposition is a Simple Structure with many Applications.

The meeting will end by a final round square table where subaltered people will express their various contrarieties, subcontrarieties and contradictions.

1.3. Scientific Committee

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2. Tutorials

Logical Oppositions: Methodologies and Applications

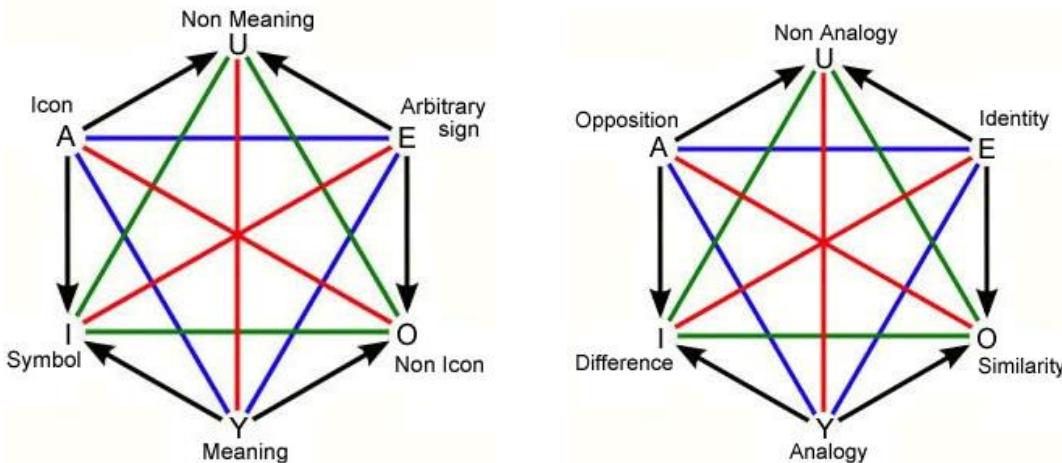
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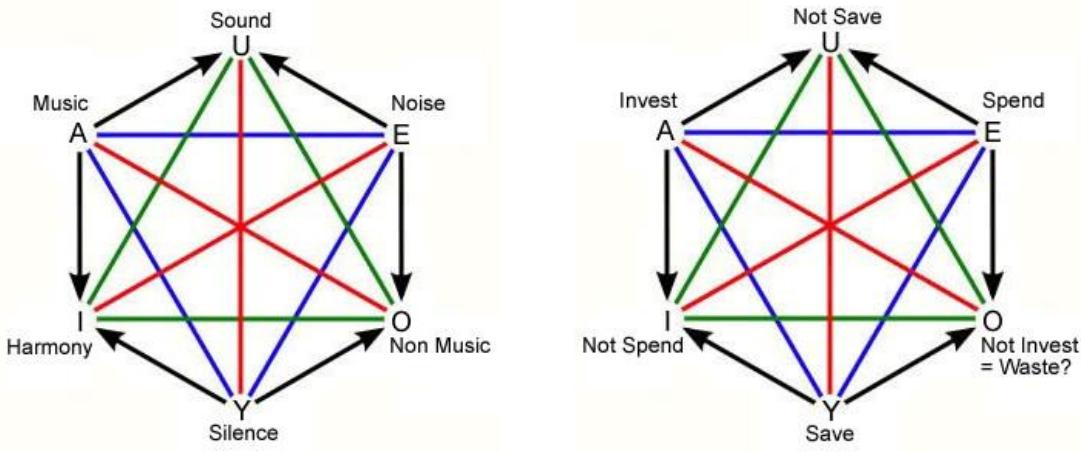
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The square of opposition arose from a particular situation: Aristotle's theory of categorical propositions. It can be generalized to many situations and its improved version, the hexagon of opposition, has even more applications. In this talk we will discuss different methodologies for the construction of figures of opposition in many different situations.

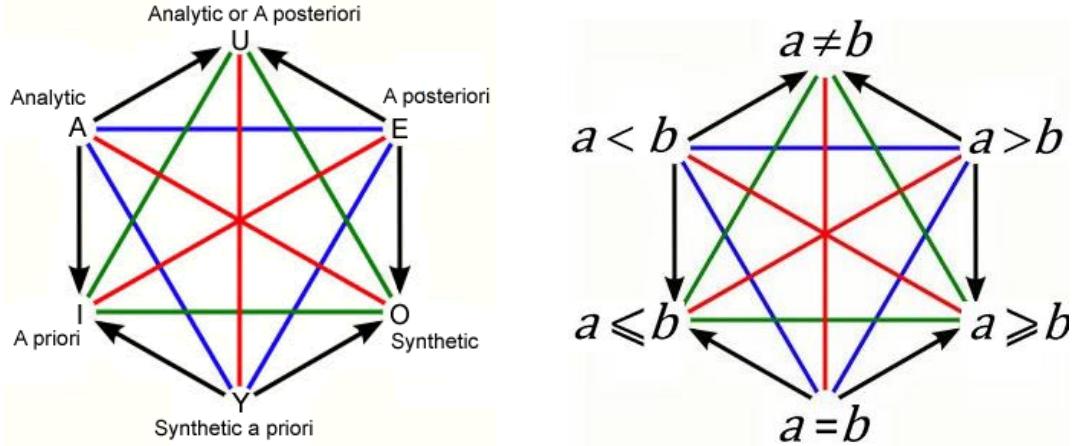
The first methodology is the construction of hexagons preserving a dichotomy and developing squares and hexagons on the basis of a dichotomy together with a subalternation. The dichotomy symbol/arbitrary sign can be preserved and with the help of the subalternation icon/icon we naturally generate a square and a hexagon. The dichotomy difference/identity can also be preserved and with the help of the subalternation opposition/opposition we can develop a square and a hexagon.



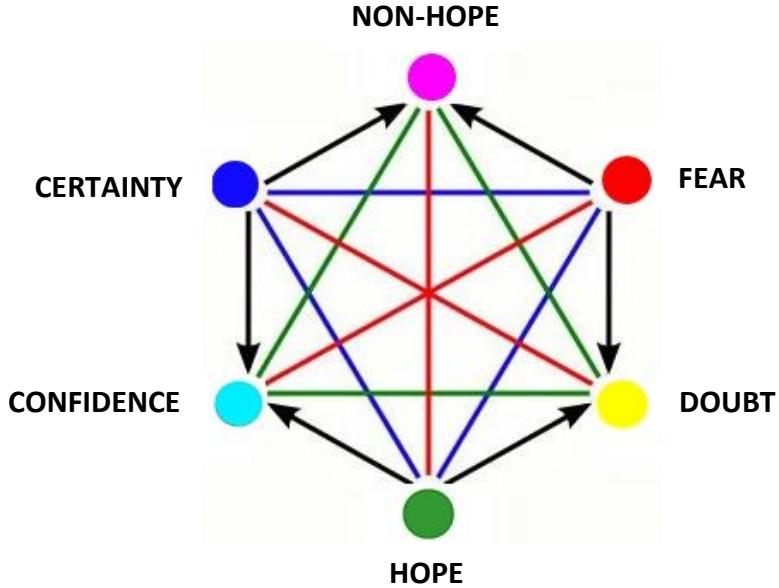
In the case of the second methodology, we start with a triangle, the result of breaking a dichotomy. The dichotomy save/spend can be broken into a trichotomy adding invest; the dichotomy music/noise can be broken into a trichotomy adding silence. The dual triangles of subcontrariety are then naturally built using contradictory axes to get hexagons.



There is a third methodology where squares are constructed by double dichotomies and hexagons are then formed over them. See the two examples below:



A fourth methodology consists in generating a hexagon from an already existing one producing a mix of the two. For example, we can consider the hexagon of colors and develop a psychic hexagon based on these colors:



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The idea of Logic in Apuleius, Boethius and other ancient commentators of Aristotle

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The school will review the idea of logic that ancient commentators of Aristotle held. Two argumentative lines can be distinguished. One takes logic to be a deductive technic. The other takes logic to be a deductive technic together with and inventive or discovery art. I argue that the ancient commentators assume these two argumentative lines without a clear or explicit merge, which produces two developments in medieval a modern tradition.

We describe these two lines. In the first, logic is identified to syllogistic. In the second, logic is also concerned to the art of finding universal premises to define o deduce universally. We argue that Boethius recognizes these two lines of development without merging or synthetize them – but keep them distinguished.

We see signs of merging them in Peter of Spain, Rudolf Agricola and especially in the young Leibniz. According, we will show how in Leibniz's *De arte combinatoria* these two lines have been unified in one ample theory of logic.

Introduction to Logical Geometry

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The central aim of Logical Geometry (LG) is to develop an interdisciplinary framework for the study of logical diagrams in the analysis of logical, linguistic and conceptual systems. Throughout history a variety of authors have constructed logical diagrams for analysing logical, linguistic and conceptual systems such as syllogistics, propositional logic, modal logic, generalised quantifiers, aspectual adverbs, colour concepts and metalogical concepts. Furthermore, on a more abstract level, several authors have studied logical and geometrical properties of various types of logical diagrams, such as the difference between Aristotelian and duality relations, the notion of Boolean closure and the relation between Aristotelian and Hasse diagrams. The first part of this tutorial will present an overview of the various **applications in logic, philosophy, linguistics and artificial intelligence** for which logical diagrams have been constructed in LG. In the second part of the tutorial, we discuss a number of abstract-logical topics related to logical diagrams, whereas in the third part, we focus on some more **visual-geometric topics**. In each of the three parts we will pay particular *attention* to the interdisciplinary variety of formal, empirical and historical perspectives adopted in LG.

As far as the **logical** applications of LG are concerned, we discuss the systems of modal logic (in particular, S5) and Public Announcement Logic. Among the **linguistic** applications we will briefly present the LG analysis of the subjective quantifiers many and few, gradable adjectives and definite descriptions. On a more **conceptual** level, LG has been applied to knowledge representation and AI, and to the perceptual field of colour theory. Underlying the analysis of this wide and interdisciplinary range of topics is the Boolean algebraic technique of bitstrings, which will be introduced at the end of the first part.

Concerning the **abstract-logical** properties of logical diagrams, LG first of all adopts an information-theoretic approach to corroborate the claim that the Aristotelian relations are hybrid between opposition relations and implication relations. A second crucial claim in LG concerns the logical independence of Aristotelian and duality relations, which relates to differences in logic-sensitivity between the two sets of relations. From the point of view of interdisciplinarity, the study of these abstract-logical topics integrates concepts from Boolean algebra, combinatorics, group theory and the philosophy of information.

As for the **visual-geometric** properties of logical diagrams, LG characterises the differences between Aristotelian and Hasse diagrams for Boolean algebras (in particular, B3 and B4) in terms of different vertex-first projections, both for 2D (hexagon) and for 3D (rhombic dodecahedron) visualisations. Furthermore, the difference between perspective and parallel projections leads to the 2D distinction between hexagons and nested triangles

in the visualisation of B3, and to the 3D distinction between the rhombic dodecahedron and nested tetrahedra in the visualisation of B4. Other crucial geometric concepts studied in LG are those of distance, central symmetry and (the complementarities between) subdiagrams. With respect to interdisciplinarity, the LG analysis of these visual-geometric topics not only employs concepts from Boolean algebra and combinatorics, but also from diagram design. More in particular, logical diagrams are argued to differ in terms of informational versus computational equivalence, and in terms of their adherence to – or violation of – such design principles as congruence and apprehension.

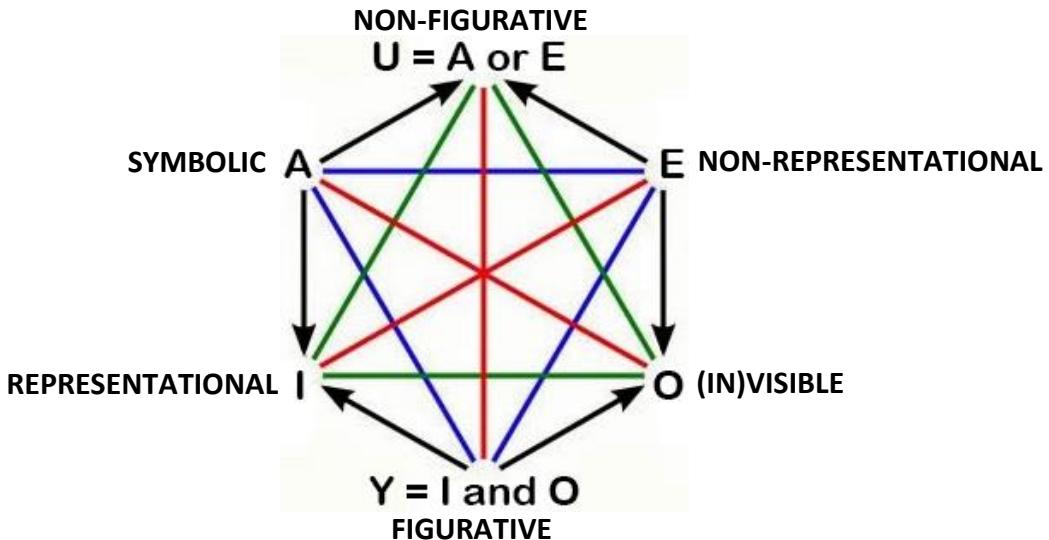
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3. Talks of Invited Speakers

Hexagon of Painting
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There are many different kinds of paintings. They are generally classified according to some *schools* (Italian renaissance painting, Chinese painting, impressionism, surrealism, etc.), *types* (portrait, landscape, still life, etc.) or *styles* (photorealism, icon, pointillism, etc.). These classifications are sometimes rather artificial or/and confuse.



In this talk we will show how the hexagon of opposition can give us a more conceptual and structural perspective regarding the classification of paintings, providing a better understanding of what has been done and what can be done.

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Reflections of the Convert's Square

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My talk takes as its starting point a converse square of opposition, which consists of the converses {ä,ë,ï,ö} of the traditional Aristotelian combinators {a,e,i,o}. (The corresponding converse determiners can be identified somewhat artificially as ONLY, NO(-LY), SOME(-LY), and NOT ONLY.) Extensionally, of course, {ä,ë,ï,ö} constitutes a square with the familiar (but converse) properties of the original square, and does not give us anything new. I will argue, though, that the merge of the two squares into a prism of oppositions may help to somewhat clarify the standard system of syllogistic reasoning.

The logical merits of adopting a prism of opposition may be deemed merely stylistic, but the widened perspective appears to be wholesome for the study of Generalized Quantifiers in Natural Language, as I will argue in the second part of my talk. The mere acknowledgement of the existence of more determiners than conservative (i.e., Aristotelian) ones only, helps in observing regular patterns in the natural use of quantifiers, which have hitherto remained undetected or without proper explanation. Some of the proposed and disputed linguistic constraints on natural language show up as pragmatic constraints on the use of certain types of determiners with rigidly identified logical properties.

While the results of the first two parts are more or less robust, the third part is more speculative. Of course, Determiner_Subject_Verb constructions in a language like English cannot just be converted, and it appears, as has been observed by Milsark, 1974, among others, that there are differences, maybe not in the truth-conditions, but in the construal of propositions according to the converse paradigm, rather than according to the traditional Aristotelian paradigm. Such differences show up most ostensibly when we consider transitive verb constructions which involve quantified direct objects. Such object positions typically need not be properly extensional (or material, Anscombe, 1965, is an early source). It turns out, or so I will argue, that quantified phrases are at home in such non-extensional contexts upon their converse construal, and can be left intentionally satisfied there. They are not at home there upon their classical construal, however, and these may then need to be intentionally accommodated.

The very subject of these investigations naturally lead me into considering the ontological status of intentional objects, and if time permits I will end sketching a concept of intentional reification without ontological commitment. Here, the concepts of reification and non-commitment are truly in the spirit of Quine, while the intentional focus is, arguably, intentionally, Quinean, too.

Squaring the unknown: from the logical square to the semiotic nonagon

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What else to say about the logical square of opposition? After more than 2000 years, the square seems inexhaustible, and yet it seems that it's almost impossible to say something about it that has not yet been said. And if Blanché's words are true, that the square captures very directly some very basic schemes of thought and rationality, this seems to be true. Beginning with this question, the aim is to present how the square is viewed from two perspectives: Greimas' text semiotic and Peirce's general semiotic.

Greimas text semiotic is primarily a semiotic of *meaning* and *signification* of texts, but it can be generalized to understand other phenomena of meaning. It is famous for its *semiotic square*, which is a model for representing oppositions of meaning that reveal in fact a inextricable mutual dependence between terms and concepts. Greimas' square draws heavily upon the traditional square of opposition, for it shows itself to be a model for understanding the not evident logical form of reality itself (being more than just a semiotic square, then).

In contrast to Greimas, Peirce had reasons to abandon the square. Placing himself into the tradition stemming from Boole and De Morgan, Peirce gives arguments to adopt a *triadic* logic. Arguing for a *reduction* thesis – thought categories are reducible to three – he gives reasons to *abandon* the square in order we are better skilled to discover something new. Nonetheless, he still commits himself with a kind of logic that keeps ontological commitments that seem to be not so distant from the Aristotelian stance. Peirce's logic, conceived as a general semiotic, has influenced several other triadic-based models, such as a recent *semiotic nonagon*, which will be briefly presented.

4. Talks of Contributing Speakers

Is there a formula to express the disparatae medieval sentences?

A positive answer

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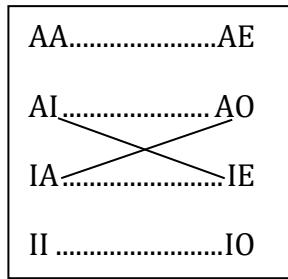
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The square of opposition consists of four sentences which show several relationships among them. The sentences can be universal or particular, affirmative or negative sentences. Universal sentences cannot be simultaneously true but can be simultaneously false; particular sentences can be true at the same time but cannot be simultaneously false. When universals are true, their particular sentences are also true but not conversely. Universal affirmative and particular negative sentences cannot be simultaneously true nor simultaneously false; the same holds for universal negative and particular affirmative sentences.

The sentences holding these relationships are known as contrary, sub contrary, subaltern and contradictory sentences. We may as well express them in the following way: sentences in the same horizontal line are contraries or sub contraries; sentences in vertical lines are subalterns being the lower sentence sub alternate to the upper; sentences placed in a diagonal line are contradictories. There are formulas which express these relationships.

The medieval octagons of opposition are constructed by resorting to an additional operator (which can be a modal operator, a quantification of the predicate or quantifying a genitive relation such as x belongs to y) which makes things more complicate. We have more complex situations since the same relationships hold for two operators. Let me put an example. In this pair of sentences "Every man necessarily argues" and "Some man possibly argues" the second is sub alternate of the first sentence regarding both quantification and modality but in this pair "Every man necessarily argues" and "Every man possibly argues" sub alternation is to be taken only regarding the modal operator. Contrary sentences and sub contrary sentences are to be considered also regarding either both quantification and modality or only one of them. Contrary and sub contrary sentences of the medieval octagon are also to be found in diagonal lines.

The medieval octagon can be depicted as a group of sentences, each one having a Subject-Predicate form. The subject is always quantified, the predicate is explicitly quantified (the first octagon), or is "modally" quantified (the second octagon combining modality and quantification), or may have a genitive quantified relation as a subject and an implicitly quantified predicate (the third octagon with sentences like, for instance, "Of every man every donkey runs" where the subject is "of every man every donkey" and "runs" is the predicate). Now, I will set this easy notation for each sentence combining the traditional letters from the square of opposition (A, E, I and O) in order to express the sentences of the octagon:



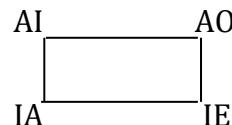
We have formulas to express the relationships:

- Contraries: $\sim(AA \wedge AE)$, in the upper horizontal line $\sim(AI \wedge AE)$, $\sim(AA \wedge AO)$ in diagonal lines.
- Sub contraries: $(II \vee IO)$ in the lower horizontal line, $(IA \vee IO)$, $(II \vee IE)$ in diagonal lines
- Subalterns: $(AA > AI, AA > IA, AA > II); (AE > AO, AE > IE, AE > II)$ in the vertical lines
- Contradictories: $\sim(AA < > IO)$, $\sim(II < > AE)$; $\sim(AI < > IE)$, $\sim(IA < > AO)$ diagonal lines forming one outer square and one inner square.

Now, the inner square of contradictories is this:



but there is no relationship for the sentences forming the square itself, besides contradictories, i.e.:



(AI and IA) are neither subalterns nor contraries nor subcontraries nor contradictories to each other; the same hold for (AI-AO, AI-IA, IA-IE, AO-IE). The “relationship” of these sentences were called *disparatae* by the medieval logicians.

My aim in this talk is to propose a formula to express the *disparatae* relationship.

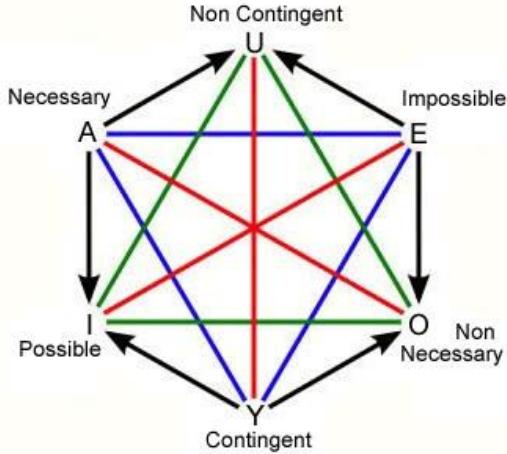
Possibility, Contingency and the Hexagon of Modalities

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During many centuries there was a confusion between two modal concepts: possibility and contingency. The situation was clarified in the mid XXth century by the hexagon of opposition, developed by Robert Blanché, making a clear distinction between something which is not impossible and something which is not impossible but not necessary.



Modern logic has focused on two modalities symbolically represented by \diamond and \Box , having some invariant features in the many systems of modal logic. “ \Box ” can easily be interpreted as necessity. “ \diamond ” is usually interpreted as possibility; however this interpretation is not so obvious from a philosophical perspective.

We will discuss this question from the point of view of the hexagon of opposition and explain how this hexagon interestingly puts forward the notion of contingency generally neglected by modern logicians.

And we will criticize the way possibility is characterized in contemporary modal logic through the diamond operator, located in the I-vertex. We will explain that it does not match with the usual notion of possibility and that this notion is better described by the Y-vertex of the hexagon of opposition. Association des Sociétés de Philosophie de Langue Française

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Reciprocity in Infinite Dimensional Oppositional Hyper Cubes

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[1] generalizes the Aristotelian notions in the Square of Opposition and builds a Cube of Opposition for some modalities while also pointing out that we may build hyper cubes of opposition of any finite cardinality for appropriate modalities. We here present infinite dimensional hyper cubes of opposition useful e.g. to capture reciprocal attitudes such as common knowledge.

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Square of Opposition, presuppositions and two kinds of negation

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In this paper I am going to show that the classical Square of Opposition is systematically ambiguous. The ambiguity stems from two kinds of factors. First, sentences come with different topic-focus articulation, and while articulating the topic of a sentence activates a presupposition, articulating the focus frequently yields merely an entailment. Thus, for instance, there are two readings of the sentence "Every *S* is *P*". Either the topic of the sentence is the subject term '*S*'. Then the sentence *presupposes* that there be some *Ss* and *merely entails* that there be some *Ps*. Or, the sentence is about *Ps*, claiming that the set of *Ps* includes all *Ss*. On this reading the sentence presupposes that there be some *Ps* and *merely entails* that there be some *Ss*. In what follows I will assume that the subject term '*S*' is the topic of the sentences in the Square, because this seems to be their intended reading.

The logical difference between a presupposition and mere entailment is this:

Q is a presupposition of R iff (R ⊨ Q) and (non-R ⊨ Q).

Thus, if the proposition *Q* is not true at a given world *w* and time *t*, then *neither R nor non-R* is true. Hence, *R* has no truth-value at such a $\langle w, t \rangle$ -pair at which its presupposition is not true.

On the other hand:

Q is merely entailed by R iff (R ⊨ Q) and neither (non-R ⊨ Q) nor (non-R ⊨ non-Q).

Hence if R is not true we cannot deduce anything about the truth-value, or lack thereof, of Q .

Thus I follow Frege and Strawson in treating survival under negation as the most important test for presupposition. If the presupposition of a sentence is not true, the sentence can be neither true nor false, because presupposition is entailed by the positive as well as negated form of a given sentence.

The other factor of ambiguity is this. There are two kinds of negation, namely Strawsonian *narrow-scope* and Russellian *wide-scope negation*. While the former is presupposition-preserving, the latter is presupposition-denying. The Strawsonian narrow-scope negated form of the sentence “Every S is P ” is “Some S is not P ”. The Russellian wide-scope negated form is “It is not true that every S is P ”. Thus, in the former case the property of not being a P is ascribed to some objects S . On the other hand, in the Russellian case the property of not being true is ascribed to the whole proposition that every S is P . I am going to prove that these two readings are not equivalent, because they denote different propositions (truth-conditions individuated up to logical equivalence). While the Strawsonian reading comes with the existential presupposition, the Russellian reading without. Yet I will also prove that in both cases the classical Square is valid, because the classical entailment relation is truth-preserving but not falsity or truth-value gap preserving.

To capture these issues, a logic of partial functions is needed. My background theory is Transparent Intensional Logic (TIL). TIL is an expressive logic apt for the analysis of sentences with presuppositions, because in TIL we work with partial functions, in particular with propositions with truth-value gaps. Moreover, procedural semantics of TIL makes it possible to define a general analytic schema of sentences associated with presuppositions, which is in particular useful in case of existential presuppositions triggered by general terms.

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Hexagons of Opposition for Statistical Modalities

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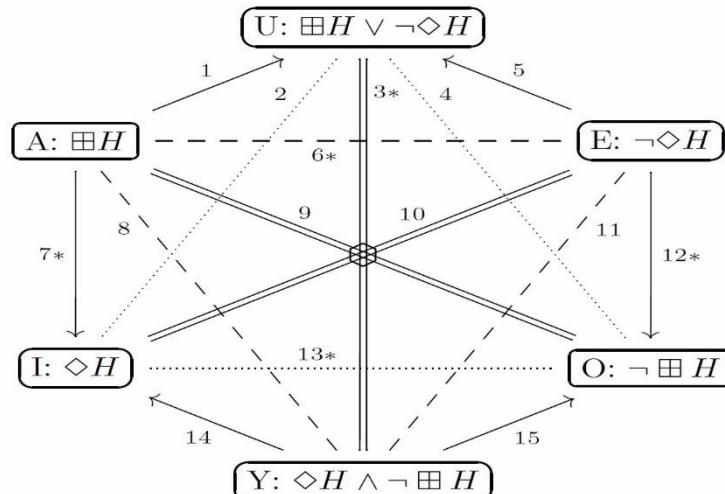


Diagram 1: Para-Consistent Hexagon of Statistical Modalities

In this article, we use the language of hexagons of opposition to analyze important logical properties of the Generalized Full Bayesian Significance Test (GFBST), as defined in [3, 5]. Furthermore, we use hexagons of para-consistent or hybrid-type opposition to analyze the interaction of statistical modal operators of different types, see [6]. For example, in the hexagon displayed at Diagram 1, the standard or “empty” symbols represent the possibilistic modal operators of acceptance and non-rejection of a statistical hypothesis, while “crossed” symbols represent probabilistic versions of the same operators. Orthodox Bayesian probabilistic truth-values are computed directly from the posterior probability measure in a statistical model, while possibilistic truth-values are computed via the probability-possibility transformation defined in [2, 8].

The hybrid nature of Diagram 1 includes para-consistent (marked with *) relations of implication (\rightarrow), contradiction ($\neg\neg$), contrariety ($\neg\perp$) and sub-contrariety ($\perp\perp$), that may fail

in paradoxical cases (ex. zero-measure hypotheses and Lindley's paradox). Finally, using the same logical setting, we analyze the philosophical importance of Blanché's Y and U vertices, represented by operators of para-non-contingency (∇) and para-contingency (Δ) of a statistical hypothesis H in the context set by the Objective Cognitive Constructivism epistemological framework, see [1, 7, 9].

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Subalternation is an opposition

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Béziau [1] has argued that subalternation is not an opposition. His main logical contentions are that that X and Y are opposed means that X and Y are "strongly different" and that no member of the pair of a subalternation can be considered as a negation of the other.

In this work, we will give two arguments for the oppositional character of subalternation. The first argument is an argument from parity and symmetry, in the lines of Moretti's [4] and Schang's [5] defenses of subalternation and the work of Humberstone [2], although the acceptance of subalternation in our proposal is smoother. The basic idea is that an opposition between A and B is an impossibility of certain combinations of semantic values for A and B. Subalternation is precisely that: B is subaltern of A if and only if it is impossible that A is designated and B is antidesignated (but it is possible that B is designated and A is

antidesignated). A and B would be strongly different in the sense that the designatedness of A implies the designatedness of B , but not the other way around. This would address Béziau's first point.

For the second one we appeal to an abstract notion of negation, namely the idea that any conditional "if A then B " is a negation of A relative to B , as it has been defined in abstract algebra (cf. for example pseudo-complements and relative pseudo-complements in Heyting algebras) and as has been insisted on from Abelian logic [3]. Thus, even if the $I(O)$ corner of a square of oppositions is not a negation of the $A(E)$ corner, it implies one: the logical negation of $A(E)$ relative to $I(O)$.

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Awareness and Creativity by Evolutive EPM

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The Elementary Pragmatic Model (EPM) was developed in the 1970s following Gregory Bateson's constructivist participant observer concept in the "second order cybernetics", to arrive to what was called "new cybernetics". Later it was applied to develop interactive psychotherapy strategies, online counseling and E-therapy. Since the beginning of the new millennium its application area has been extended to other disciplines and even to engineering problems like user modeling, constraint requirements elicitation, software creativity and adaptive system design and development. Classic EPM associated Boolean Algebra $B_3 \equiv \varphi(\{1, 2, 3\})$ can be represented LTR (Left-To-Right) by cube C_3 with its bitstring decoration in R^3 . C_3 can be thought as an extension of basic Aristoteles's "Square of

Oppositions”, including Jacoby-Sesmat-Blanché Hexagon σ_3 . Quite recently, EPM intrinsic Self-Reflexive Functional Logical Closure contributed to find an original solution to the dreadful IDB (information double-bind) problem in classic information and algorithmic theory. The IDB problem is just at the inner core of human knowledge extraction by experimentation in current science. EPM is even a high didactic versatile tool and new application areas are envisaged continuously.

In turn, starting by classic EPM approach through evolutive hypercube geometric algebra, this new awareness has allowed to enlarge our panorama for neurocognitive system behavior understanding, and to develop information conservation and regeneration systems in a numeric self-reflexive/reflective evolutive reference framework. In this talk we propose a notation that goes beyond a format distinction and constructed with the purpose to facilitate inferences either on a diagrammatic representation, or a lexical one. The latter particularly allows operations on complex propositions within hypercube with more than three dimensions, mentally difficult to imagine. EPM extension as “Evolutive Elementary Pragmatic Model” (E²PM) represents a contribute to current modeling and simulation, offering an example of new forms of evolutive behavior by inter- and trans-disciplinarity modeling (e.g. strategic foresight, uncertainty management, embracing the unknown, etc.)

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**An intuitionistic version of the square of opposition in the topos approach
to quantum mechanics**

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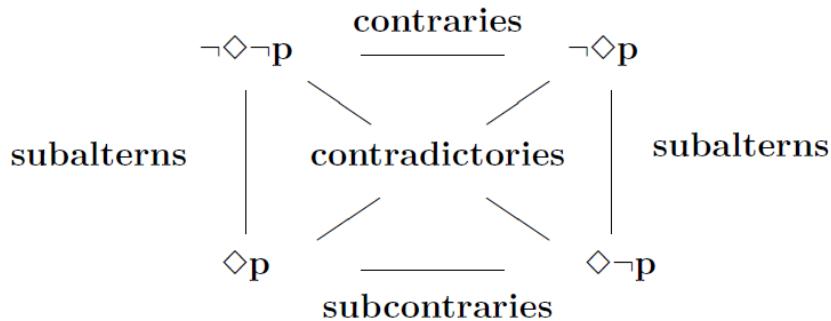
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The traditional Square of Opposition expresses the essential properties of monadic first order quantifiers \forall and \exists taking into account the categorical propositions. This notion of opposition admits an algebraic counterpart when the quantifiers are considered as modal operators acting on a Boolean algebra. In this way, the Square of Opposition represents relations between certain terms of the language of Boolean algebras. The algebraic structure of these relations is shown in the following scheme, sometimes called the Modal Square of Opposition:



The modal version of the square is very useful since it allows us to translate the notion of opposition into other logical systems. Indeed, by changing the underlying Boolean structure by a different structure, we obtain new interpretations of the square. For example, using a modal extension of orthomodular quantum logic [1, 2], a version of the square related to quantum logic was established in [4]. This particular notion of opposition describes properties of non-contextual quantum systems. In the last years several approaches based on category theory have been used to describe quantum systems. In the topos approach to

quantum mechanics [6, 7], the quantum analogue of classical phase space is captured by a presheaf called spectral presheaf. Thus, properties about a quantum system are encoded in an algebra of sub-objects of the spectral presheaf. This algebra naturally endows an intuitionistic structure to the properties about the quantum system. In the present work, by considering a modal extension of this intuitionistic structure introduced in [3], we present a new version of the square of opposition. The notion of opposition, in this variant of the square, describes properties about deductive systems related to the intuitionistic approach to quantum systems.

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The Hypercube of Dynamic Opposition

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The goal of this talk is to present an opposition structure between formulas of Propositional Dynamic Logic (PDL henceforth). This structure is called Hypercube of Opposition of Programs and his main function is to show the varieties of resulting opposition operations by joining two extensions of PDL, in specific this structure shows 120 theorems with four exceptions, the called disparatae. The aforementioned formulas highlight because they are not susceptible to meet any of the four traditional opposition operations, i.e. contradiction, contrariety, subcontrariety, and subalternation.

The novelty of our approach lies in the following features: 1) the language of the PDL extension admits two kinds of negations one of atomic programs and other for formulas, 2) there is a predominance of subcontrariety opposition in the structure, and 3) is possible to generate derivate negations both, for negation of programs and for negation of formulas based on the combination of a modal operator and the aforementioned negations. The latter issue has been studied for example in [1, 2, 3, 4, 5]. In this case the novelty lies in studying the same phenomenon but with the negation of programs.

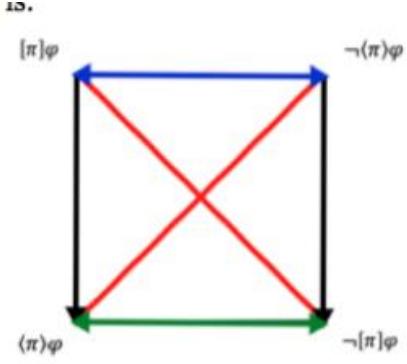


Fig. 1 SQUARE OF PROGRAMS

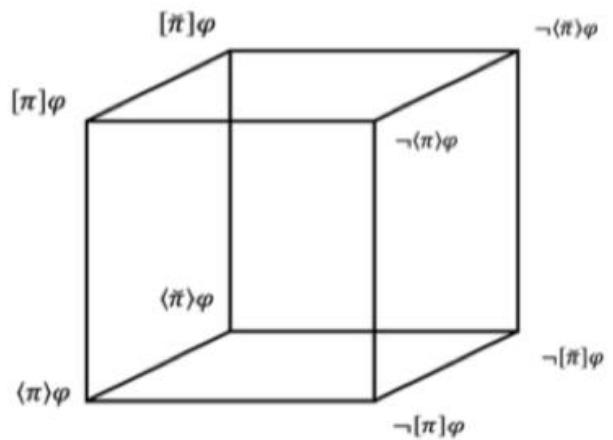


Fig. 2 CUBE 1

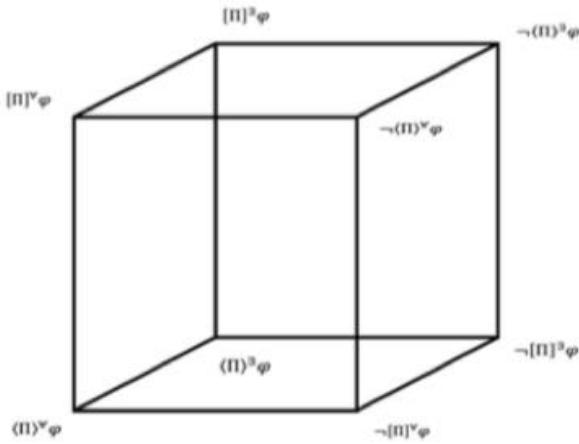


Fig. 3 CUBE 2

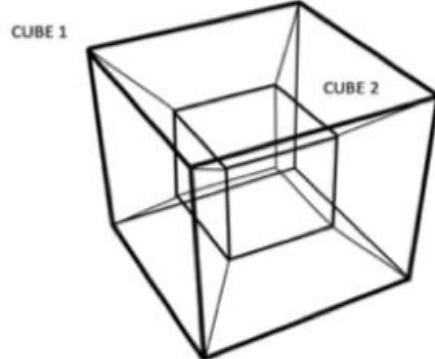


Fig. 4 HYPERCUBE

We begin with the dynamic version of the modal square of opposition as shown in Fig. 1, in this sense both the square and PDL are the base of our analysis. The Hypercube is presented on an extension of PDL that we call Propositional Dynamic Logic with dynamic modalities and negation of atomic programs ($\overline{\text{PDL}_Q}$ for short), that has as sublogics PDL with negation of atomic programs (PDL[¬] as shown in [4]) and PDL of dynamic modalities (PDL_Q as it presented in [6]). In this sense we extend the square in two different ways, by one side towards a cube of opposition of atomic programs as shown in Fig. 2; and by the other side to a cube of dynamic modalities as shown in Fig. 3. Each cube belongs to a different fragment of $\overline{\text{PDL}_Q}$ and, because of that, to present the union of the opposition structures indirectly we analyze the union of the aforementioned logics.

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Relating logics – axiomatizations and extensions

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As we know usual Boolean connectives capture only truth-value relationships between sentences. For example, material implication expresses a piece of information that its antecedent is false or its consequent is true. However, in natural languages we establish additional relationships (e.g. causal, temporal, analytical etc.), so we would like to have, for example, an implication, a conjunction, etc., that express the following ideas:

- a causal relationship:
 - If A , then it causes B .
 - A and it caused B .
- a temporal relationship:
 - If A , then next B .
 - A and next B .
- an analytical relationship:
 - If A , then analytically B .
 - A and analytically B .

Relating logics (in short: RL) are new kind of relevant logics, where phenomenon of relevance is defined on the level of new, non-Boolean connectives. By such connectives, called *relating connectives*, we can describe a mechanism of relating formulas and in this way cover formally the problem of relationships of sentences. Moreover, in model of RL some intensional relationship is cached by a binary relation determined on the set of formulas of the language of RL. With such simple and yet flexible tool we are able to introduce new solutions for many important philosophical problems.

A main scientific issue we aim to present in the paper is an axiomatization of the smallest RL. The logic we call RF. Moreover, we would like to introduce a new modal notion *strict*

relating implication, which is an improvement of usual strict implication. Due to the new notion we omit some paradoxes that appear in the context of strict implication, at the same time preserving good properties of strict implication. And last but not least we would like to introduce a square of oppositions that expresses some logical relationships between relating connectives on the ground of RF.

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Modal Syllogistic de re: Semantics, Tableaus, Estimations and Square of Oppositions

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Modal syllogistic *de re* deals with modal categorial propositions which are interpreted as *de re*. We have twelve kinds of propositions: a) four classical categorial propositions (without modalities), b) eight modal propositions such as:

- | | |
|------------------|--|
| $Sa^{\Box}P$ | Every S is necessarily P . |
| $Si^{\Diamond}P$ | Some S is possibly P . |

As we know, some logicians (Aristotle, Jan Łukasiewicz and others) formulated foundations for different systems of modal categorial syllogistic. On the other hand, we start our research from semantic aspects of modal categorial propositions.

In our approach, we do not use possible worlds' semantics. An initial model of modal categorial propositions language is any quadruple $M = (D, f^{\Box}, f, f^{\Diamond})$, where D is a set and f^{\Box} , f , f^{\Diamond} are functions defined on the set of terms into the powerset of D , such that $f^{\Box}(X) \subseteq f(X) \subseteq f^{\Diamond}(X)$, for any term X .

The models enable us to give a reasonable interpretation. For example, for all terms X, Y we put:

- (a) $M \models Xa^{\Box}Y$ iff $f(X) \subseteq f^{\Box}(Y)$
- (b) $M \models Xi^{\Diamond}Y$ iff $f(X) \cap f^{\Diamond}(Y) \neq \emptyset$

and similarly for other categorial propositions.

For the interpretation, we introduce a tableau system that corresponds to the given interpretation of modal propositions. We show the *squares of opposition* they form – we write *squares*, since those figures usually are other sorts of polygons. We should remember

that in the case of modal categorial propositions there are more than only one possible square, since our language consists of twelve kinds of propositions.

The last part of our paper consists of a presentation of two kinds of estimations we present. The first estimation allows to predict a maximal cardinality of countermodel for a given argument, while the second estimation is syntactic – thanks to it we can predict maximal length of branches that terminate a tableau proof.

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Aristotle's other squares

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The idea of this talk is to show that Aristotle works on other logical diagrams in his logical writings. One is in *De Interpretatione* 10, the other is in *De Interpretatione* 13. Both squares are also referred to in Aristotle's *Prior Analytics* I,3 and I,46, so the aim is to compare these treatises and produce an idea of the logical doctrines behind. These diagrams are related to Aristotle's logic of modal oppositions and semantic relationships between indefinite and privative propositions. I take the opinions of the first commentators to discuss and project the transmission line of these both issues.

A characterisation of some Béziau-like counterparts of quasi-regular modal logics

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In the context of modal logics one standardly considers two modal operators: possibility (\Diamond) and necessity (\Box) (see for example [4]). If the classical negation is present these operators can be treated as inter-definable. However, negative modalities ($\Diamond\neg$) and ($\Box\neg$) are also considered in the literature (see for example [7, p. 497], [3, p. 300], [2] and [1]). Both of them can be treated as negations that are opposite to respective positive modal operators. Its meanings rely on a modal logic in which respective translations are applied.

In [1] a logic Z has been defined on the basis of the modal logic S5. Z is proposed as a solution of so called Jaśkowski's problem (see also [8]). The only negation considered in the language of Z is 'it is not necessary'. Following a suggestion given there we can consider

other modal logics instead of S5 obtaining in this way Béziau logics where again the only negative operators are modal ones. The initial correspondence result between standard modal logic S5 and its counterpart, Béziau logic Z has been generalised for the case of normal logics leading in particular to soundness-completeness results (see [9, 10]). It has been shown, that there is a general way to go from completeness results for normal modal logics to completeness results for respective Béziau logics. In [11, 12] some partial completeness results for non-normal case are given, in particular in [12] a completeness theorem for the case of the counterpart of a quasi-regular (see [14]) logic is proposed. In [13], more general correspondence result was proposed for regular logics (for syntax and semantics for standard regular logics see [6, 5, 14]), where both modal negations ('it is necessary that not' and 'it is not necessary') have been used.

A question arises to which extent, similar, more general results can be obtained for quasi-regular logics. Thus, in the present paper we consider correspondence between quasi-regular modal logics and respective Béziau-like logics.

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Caramuel’s Theory of Opposition

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In 1654 [1], the Spanish philosopher and theologian Juan Caramuel y Lobkowitz published his *Theologia rationalis*, which contains many interesting observations and innovations relevant for the Square of Opposition.

First, in addition to the usual theory of opposition of propositions, Caramuel also investigates the opposition of terms, (e.g., ‘Human’, ‘Brute’, ‘Not brute’, ‘Not human’).

Second, besides the traditional opposition of the categorical forms (‘Every *S* is *P*'; ‘No *S* is *P*'; ‘Some *S* is *P*'; ‘Some *S* is’nt *P*'), Caramuel takes into account the opposition of modal propositions (e.g., ‘Necessarily *q*’, ‘Impossibly *q*’, ‘Possibly *q*’, ‘Possibly not-*q*’), and of exclusive propositions (e.g., ‘Only *S* are *P*’; ‘Only *S* are not-*P*’; ‘Not only *S* are not-*P*’; ‘Not only *S* are *P*’).

Third, according to [2], Caramuel’s most important logical innovation consists in the development of a so-called ‘oblique logic’ which deals with quantified relational propositions (e.g., ‘Every *S* loves every *P*’, ‘No *S* loves no *P*’...). This gives rise to another interesting expansion of the traditional square of opposition.

Fourth, Caramuel develops a theory of ‘Transsubstantiation’ and ‘Transfiguration’ which turns out to be a powerful extension of the traditional theory of conversion. Within this theory Caramuel considers propositions whose predicate-part explicitly contains quantifiers like ‘every’, ‘some’ or ‘no’ (e.g., ‘Every *S* is every *P*’, ‘No *S* is every *P*’...). As was shown in [3], however, Caramuel failed to develop a general semantics for these propositions. Therefore, contra the judgment of [4] and [5], Caramuel may not be accredited with having invented the theory of the ‘Quantification of the Predicate’.

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Logic and Rhetoric – about the Role of Chiasmus in Shakespeare's Tragedy Hamlet, Prince of Denmark

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In the 50s and 60s of the 20th century representatives of structural linguistics were trying to implement in the area of humanities methods and concepts borrowed from exact science. Algirdas J. Greimas [1,2] reckons that every text has an unchangeable structure which could be represented by the so called semiotic square (Carré sémiotique), introduced by him.

The decay of the structuralism does not mean that literary texts could not be explored by exact methods. Shakespeare's tragedy Hamlet [3] could be pointed out as an example. It contains many hidden or explicit paradoxes. Some of them have to be analyzed by the means of three-valued logic of action.

In this proposal, I would like to point out the meaning of rhetoric figure chiasmus (from the form of Greek letter "χ"). It depicts the relations between four interconnected parts of sentence according scheme ab/ ba, so that the first one is coordinated to the fourth, and the second – to the third.

Shakespeare uses chiasmus at the very beginning of his play. Hamlet makes Horatio and Marcellus swear: "Never make known what you have seen tonight." They answer:

| | |
|---------------------------|---|
| Horatio: In faith, | Marcellus: Nor I, in faith. |
| my Lord, | |
| not I. | |

In Shakespeare's play [3] chiasmus is related not only to the performative act of swearing. Polonius considers it as a "foolish figure". He says to the Queen: "Madam, I swear I use no art at all. That he (Hamlet) is mad, 'tis true: 'tis true 'tis pity. And pity 'tis 'tis true."

Although chiasmus is "foolish figure", Shakespeare uses it in order to show how the hidden secret could be revealed by the help of language. The liar cannot hide his guilt all the time. Shakespeare uses different kind of chiasmus in order to show how the murder could be exposed.

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Square and Hexagon of Opposition of “A-Priori Knowledge” and “Empirical One” (Eliminating an Impression of Logic Contradiction between Leibniz’ and Gödel’s Statements)

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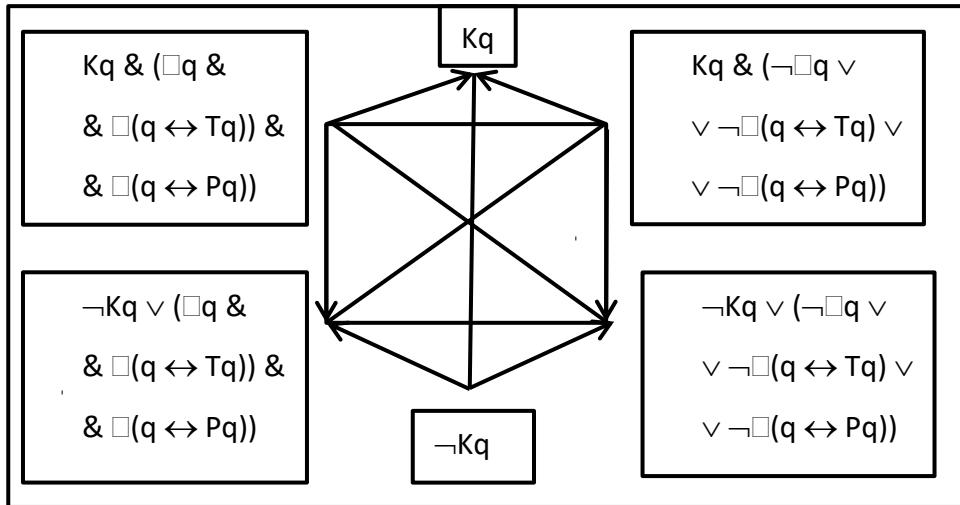
In 1686, in "Generales Inquisitiones de Analysi Notionum et Veritatum" (Lat.), G.W. Leibniz had proclaimed (several times) the universal principle that every true proposition is provable ([1], pp. 371, 387, 389]). This principle of Leibniz makes an impression of contradiction with the incompleteness theorems of K. Gödel. In spite of this impression below I submit logically consistent combining Leibniz' and Gödel's statements in one conceptual scheme of epistemology (both men are right but in different relations).

Let symbols Kq , Aq , Eq , Tq , Pq stand for epistemic modalities: "(person has) knowledge that q "; "(person has) a-priori knowledge that q "; "(person has) empirical knowledge that q ", "it is true that q ", "it is provable that q ", respectively. The symbols $\&$, \vee , \leftrightarrow , \neg have classical logic meanings. The sign \Box stands for the alethic necessity; q – a proposition. In 2015 I had submitted the following definitions of the compound epistemic modalities Aq and Eq .

DF-1: $Aq \leftrightarrow (Kq \& (\Box q \& \Box(q \leftrightarrow Tq) \& \Box(q \leftrightarrow Pq)))$.

DF-2: $Eq \leftrightarrow (Kq \& (\neg\Box q \vee \neg\Box(q \leftrightarrow Tq) \vee \neg\Box(q \leftrightarrow Pq)))$.

If the definitions are accepted, then the logic connections among the epistemic modalities Kq , Aq , Eq , $\neg Aq$, $\neg Eq$, $\neg Kq$ are represented by the below square and hexagon of opposition.



If the applicability domain of the mentioned principle of Leibniz is defined as (or reduced to) the sphere of a-priori knowledge exclusively, then the logic contradiction is eliminated, as in this case, in accordance with the incompleteness theorems of Gödel and the above definitions DF-1, DF-2, the knowledge of arithmetic belongs to the sphere of empirical one.

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Extension in The Port Royal Logic

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Although the notion of extension is central to the definition of truth for categorical propositions in the Logic of Arnauld and Nicole, its exact meaning is obscure, as noted by Kneale and Kneale in their important history [2]. Commentators agree that a term's extension consist of its inferior ideas, but there is dispute about what inferior means here. Some like [3] hold an intensional interpretation that an idea's inferiors are all those ideas defined in its terms, making the truth of a universal affirmative a matter of the conceptual inclusion of the predicate by the subject. In this paper I argue for a referential interpretation in term of signification: idea A is included in the extension of idea B iff all the objects that A signifies B also signifies. I argue that the referential interpretation is required by, and that the intensional interpretation is inconsistent with, various features of the Logic's metatheory including the truth-conditions of categorical propositions (as in [1]), the Logic's distinction between contingent and necessary truth, and its doctrine of false ideas.

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Oppositions in Pure versus Applied Logics

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An interesting perspective on the traditional opposition between pure and applied sciences is provided by Pasteur's Quadrant. Pasteur's quadrant states the thesis that scientific research tries to both push back the frontiers of knowledge, while at the same time also tries to use the new understanding gained for improving human life. Pasteur's quadrant may be viewed as an attempt to deconstruct the traditional opposition between pure science versus applied science. We use the framework of the quadrant to examine the opposition between pure and applied logic, using the insights gained in work on automated proof-checking.

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The square of opposition: Four “colors” enough for the “map” of logic

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Intrigue (a few questions): Why square? Why four? What is the common in the following facts?

- 1) The square of opposition.
- 2) The “letters” of DNA.
- 3) The number of colors enough for any map.
- 4) The minimal number of points, which allows of them not be always well-ordered.

The number of entities in each of the above cases is four though the nature of each entity seems to be quite different in each one.

Prehistory: The first three share (1-3) being great problems and thus generating scientific traditions correspondingly in logic, genetics and mathematical topology. However, the fourth one (4) is obvious: triangle has not diagonals, quadrangle is just what allows of

its vertices not to be well-ordered in general just for its diagonals. Thus the limit of three as well as its transcendence by four seems to be privileged philosophically, ontologically, and even theologically: It is sufficient to mention Hegel's triad, Peirce's or Saussure's sign, Trinity in Christianity, or Carl Gustav Jung's discussion about the transition from Three to Four in the archetypes in "the collective unconscious" in our age.

Thesis: The base of all cited absolutely different problems and scientific traditions is just (4). Thus, the square of opposition can be related to those problems and interpreted both ontologically and differently in terms of the cited scientific areas and in a few others.

Arguments in favor of the thesis:

(1) Logic can be discussed as a formal doctrine about correct conclusion, which is necessarily a well-ordering from premise(s) to conclusion(s). To be meaningful, that, to which logic is applied, should not be initially well-ordered just for being able to be well-ordered as a result of the application of logical tools. (2) Consequently the initial "map", to which logic is to be applied, should be "colored" at least by four different types of propositions, e.g. those kinds in the square of opposition. They are generated by two absolutely independent binary oppositions: "are – are not" and "all – some" thus resulting exactly in the four types of the "square". (3) Five or more types of propositions would be redundant from the discussed viewpoint since they would necessary iff the set of four entities would be always well-orderable, which is not true in general. (4) Logic can be discussed as a special kind of encoding namely that by a single "word" thus representing a well-ordered sequence of its elementary symbols, i.e. the letters in its alphabet. The absence of well-ordering needs at least four letters to be relevantly encoded just as many (namely four) as the "letters" in DNA or the minimal number of colors necessary for a geographical map. (5) The alphabet of four letters is able to encode any set, which is neither well-ordered nor even well-orderable in general, just to be well-ordered as a result eventually involving the axiom of choice in the form of the well-ordering principle (theorem). It can encode the absence of well-ordering as the gap between two bits, i.e. the independence of two fundamental binary oppositions (such as both "are – are not" and "all – some" in the square of opposition).

Luhmann's Analysis of Historical Meaning and the Square of Opposition

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Among the various aspects of human society analysed by Niklas Luhmann (1927-1998) in his sociological systems theory are the approaches by which it would be appropriate to "go beyond a naïve relation to our own history". Luhmann points out that, apart from accessing the historical past by means of a methodological or an epistemological perspective (i.e. "to ask under what conditions we can come to know it"), the respective historical meaningfulness is to be considered as well. It is, accordingly, Luhmann's intention to "show how social systems constitute time, temporal horizons, and specific interpretations of what is temporally relevant". He argues that "historical events [...] are viewed as 'relevant' or meaningful not because they are purely factual and not merely

because of the sequence in which they happen to occur, but rather because they can be understood as having been selected from an array of other possibilities", i.e. being conceptualised within the scope of specific temporal horizons. It follows, therefore, that is it possible to distinguish and observe, apart from the factuality of past events, various ways how present historians (or present societies) include or exclude specific past events when referring to the past. Of special significance within such a view upon the historical past are the various possibilities of how any respective earlier/later distinction is conceptualised and actually instantiated, as e.g. modern historical research is "concerned with past presents, not with the presence of the past" (thereby aiming at an objective analysis of a specific past, clearly being distinguished from the present). In order to further analyse Luhmann's approach towards history I intend, in my paper, to rephrase Luhmann's distinction as an application of the square of opposition, namely by choosing the following steps in such a re-wording of Luhmann's concept:

- 1) Every past event is included within the present (i.e. unlimited presence of the past),
- 2) No past event is within the present (i.e. the various pasts characterized as past presents),
- 3) Some past event is included within the present,
- 4) Some past event is not included within the present.

Providing thereby, in a first step, a detailed description of Luhmann's distinction, and followed by its rephrasing in terms of the square of oppositions, I shall then conclude by analysing implications of this operation, i.e. with regard to the possible limits hereof as well with regard to further conclusions to be drawn (in respect of historical research as well as of the temporal concepts as developed by Luhmann in his systems theory).

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Transreal truth valued square of opposition

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In [6] we propose a total semantics. This semantics is based in the set of the transreal numbers, \mathbb{R}^T . Transreal numbers were proposed initially in order to be applied to computer science [1], however they are now being used in logic too [2, 6, 7]. Transreal numbers are made up of the real numbers, together with three, definite, non-finite numbers: negative infinity, $-\infty := -1/0$; positive infinity, $\infty := 1/0$, and nullity, $\Phi := 0/0$. Transreal arithmetic is total, in the sense that the fundamental operations of addition, subtraction, multiplication and division can be applied to any transreal numbers with the result being a transreal number. In particular division by zero is allowed [3]. By virtue of the totality of transreal arithmetic, \mathbb{R}^T is chosen to model the total semantics. In Figure 1 the transreal line is shown.

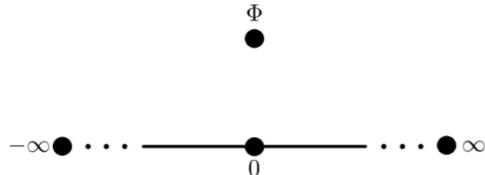


Figure 1. Tranreal line

The axis is scaled to allow all real numbers to be laid out in the figure. Negative infinity lies to the left of the real line. Similarly, positive infinity lies to the right of the real line. Nullity lies off the direction of the real line. In [6] we define \mathbb{R}^T as the set of truth values and define, in an adequate way, the connectives negation, conjunction and disjunction. We prove that negative infinity models the classical truth value False, positive infinity models the classical truth value True, the set of real numbers models fuzzy values, zero models a paraconsistent value which is equally False and True, and nullity models an indeterminate value which is neither False nor True.

Here we propose a square which extends the square from the Belnap's four-valued logic [4]. In Belnap's logic the truth values set is given as $\{F, T, \delta, \gamma\}$ where F is False, T is True, δ is False and True, and γ is neither False nor True. That square was shown in [5], Figure 2.

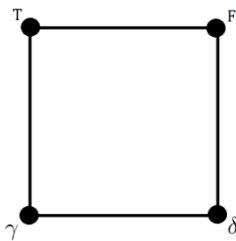


Figure 2. Four-valued square

Our **transreal truth valued square of opposition** is a square made from total semantics of transreal numbers, Figure 3.

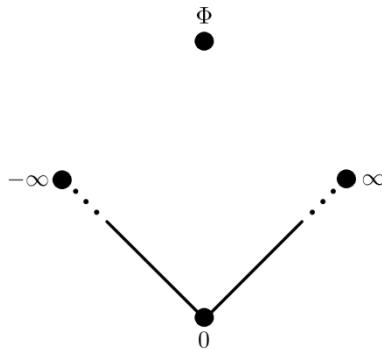


Figure 3. Transreal truth valued square of opposition

This square is continuous at some points and discontinuous at other ones. Transreal numbers $-\infty$, 0 , ∞ and Φ , in this order, are the vertices. The side between the vertices 0 and $-\infty$ is made of a semi-straight line which begins at 0 and goes indefinitely with all negative real numbers toward to $-\infty$. In same way the side between the vertices 0 and ∞ is made of a semi-straight line which begins at 0 and goes indefinitely with all positive real numbers toward to ∞ . The side between the vertices $-\infty$ and Φ is made of a gap. In same way the side between the vertices ∞ and Φ is made of a gap. In this square the vertices are not the only truth values, but the continuous part of the sides are also truth values. Note that $-\infty$, 0 , ∞ and Φ play the role of Belnap values F , δ , T and γ respectively, the side between 0 and $-\infty$ plays the role of fuzzy value of falsehood and the side between 0 and ∞ plays the role of fuzzy value of truthfulness.

In transreal truth valued square of opposition, the opposite values are the intersections of the square with lines parallel to the diagonal between $-\infty$ and ∞ , Figure 4.

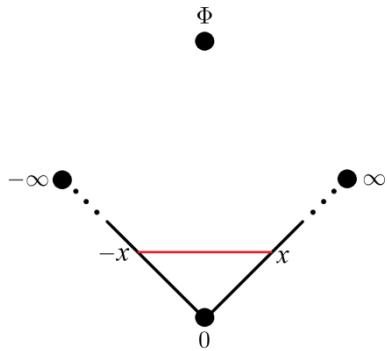


Figure 4. Opposite values

In this way, symmetrical real numbers are opposites, $-\infty$ and ∞ are opposites, 0 is opposite to 0 and Φ is opposite to Φ . This is according to trasnreal negation defined in [6], where $\neg(x) = -x$ (transreal arithmetic gives $-0 = 0$ and $-\Phi = \Phi$). Let us consider the triangle $-x0x$ whose hypotenuse is $-xx$. This triangle represents the “paraconsistent negation”. The “paraconsistent triangle” could be increased, and in the limit, when $-x = -\infty$ and $x = \infty$, we have the “classical triangle” $-\infty\infty$. In this way, negation is a logical operation that “diagonalizes” truth values. Every logic that has contradiction could

be seen as an operation in a triangle, such that operation goes from the position x to the antipodal position $-x$. Contradiction must be defined in a triangle whose origin is a *dialetheia* – a true contradiction.

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Structural Theory of Information Systems as Meta-Theory of Logic: Square of Opposition for Information

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Philosophy of logic is coming of age with the recognition of the lack of commonly accepted answer to the question "What is a logic?". [2] consider this lack of formal answer "embarrassing". The same sense of embarrassment or even scandal is present in other maturing philosophies of fundamental concepts defining domains of research such as "information", "computation", "complexity". However, it is not the lack of definite answers embarrassing, but the lack of recognition that there is no unique, ready answer to such questions possible. In each case there are some examples of logics, instances of what is considered information, standard model of computation in the form of a Turing machine whose uniqueness is questioned, or experience of difficulties to deal with some complex systems and the expectation that one formally defined concept can unify varieties of examples. But the answer to the question "What is...?" can be given only in reference to multiple constructions of theories based on competing definitions, not by a consensus of

sages or contest of popularity. After all, in case of logic we can expect, in agreement with Béziau's anticipation [1] that many other forms of logic can be introduced in the future.

The present paper is a contribution to the discussion of the question "What is a logic?" in terms of structures describing information systems and their mutual correspondences originally outlined in earlier papers of the author [3, 4]. In this context Square of Opposition acquires a new perspective. Of course, the concept of information used here is not the one, unique, embraced by everyone (no such concept exists, nor is possible), but it is a result of the individual choice of the author for the purpose of theoretical description unifying majority of previously considered instances of information. In this approach, the existing examples of logical systems (syllogistic, propositional logic, predicate logic) are special instances of information systems defined by structures identified in formally defined languages.

The key concept unifying these instances is Tarski's consequence operator C_n , i.e. a transitive closure operator of finite character on the set of sentences with additional conditions imposing consistence of the closure operator with particular structure of a linguistic system involved in each case. For instance, one of the additional axioms imposed on the consequence operator to describe propositional logic makes the subsets closed with respect to the consequence closure operator (C_n -closed subsets) also closed with respect to the rule of modus ponens. In the algebraic form this restricts the C_n -closed subsets to filters in the Boolean algebra generated by logical operations.

The main problem of philosophy of logic (as well as philosophy of information) has been always question about the meaning of meaning, i.e. semantics for logical or informational systems, which in turn generates questions regarding validity of reasoning. Typical approach is an escape from the question by involving a formal valuation (true – false) or by the use of metalanguage to define when a sentential form is satisfied and then reducing the question of meaning to models of theory in terms of the set theory.

In the present paper, meaning is a correspondence (homomorphism) between logical systems (defined by closure operators of logical consequence) and information systems constituting reality (also defined by closure operators, but of less restricted type). Using this approach to semantics of information, we can identify Square of Opposition for Information extending the range of information systems beyond that in earlier paper of the author [4].

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Lateral Activation and Lateral Inhibition and Two Squares of Opposition

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It is known from neurobiology that there are two different synaptic effects: (i) excitatory effect (depolarization) that increases the membrane potential to make neuron more negative and to decrease the likelihood of an action potential and (ii) inhibitory effect (hyperpolarization) that decreases the membrane potential to make neuron more positive and to increase the likelihood of an action potential. So, lateral activation (LA) is the structuring of a neural network so that neurons activate their neighbours to decrease their own responses and lateral inhibition (LI) is the structuring of a neural network so that neurons inhibit their neighbours in proportion to their own excitation. In other words, the more neighbouring neurons stimulated, the less strongly a neuron responds and the fewer neighbouring neurons stimulated, the more strongly a neuron responds. Thus, in the LI and LA modes we prefer items in a different manner: in LI we compare a few items with their possible (numerical) comparability to formulate explicit preference relations, while in LA we compare many items and put forward general estimations without details and without scoring functions. There are populations which behave as a distributed network, capable of responding to a wide range of spatially represented stimuli, for example, colonies of ants or fungi have such a behaviour. In their behaviours we can observe effects of neural networks with LA and LI mechanisms. We have shown experimentally that effects of LA and LI are detected in the plasmodium of *Physarum polycephalum* (please see <http://www.phychip.eu>). Thus, in the plasmodium approximation of the neural response, LA is represented by chemoattraction with splitting of plasmodium and LI is represented by chemoattraction with fusion of plasmodia. Both are the two different ways of distribution density of protoplasmic network, i.e., the two different ways of the plasmodium concentration in its networking. Fusion and splitting are key motions in the plasmodium propagation as well as LI and LA are key reactions of neural nets to stimuli. Notably, the fusion and splitting of plasmodium can be interpreted syllogistically. The spatial deducing according to fusion is formalized as a spatial version of Aristotelian syllogistic and the spatial deducing according to splitting is formalized as a spatial version of performative syllogistic defined in [1, 2]. In both syllogistic formalizations, all data points are denoted by appropriate syllogistic letters as attractants. These attractants are scattered at different places and the plasmodium tries to occupy them. A data point S is considered empty if and only if an appropriate attractant denoted by S is not occupied by the plasmodium. We have syllogistic strings of the form SP with the following interpretation: 'S and P are comparable

positively', and with the following meaning: SP is true if and only if S and P are neighbours and both S and P are not empty, otherwise SP is false. We have proven that the conventional square of opposition is related to LA and the unconventional square of opposition proposed in [1, 2] is related to LI. In the second case pairs cannot be measurable by numbers, because they are not mutually related by the quantitative ordering.

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Properties fair or: Realms of reality in Aristotle

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In my contribution I shall deal with the different levels of existence of some entities constituting Aristotle's interpretation of reality. In my analysis I shall first take some suggestions from Aristotle's defence of the principle of contradiction in Metaphysics Gamma in order to show the value of the principle of contradiction as the structural formula of the reality and in order to show the relationships between principle of contradiction and properties; then I shall take into consideration Aristotle's examination of universals in the Posterior Analytics in order to survey the relationships between universals and the feature of belonging from necessity, on the one side, and in order to link the positions of the Posterior Analytics to both the essential properties and to the properties which derive from the essence, on the other side. Thereafter I shall connect the characteristics of the universals in the Posterior Analytics with the modal syllogisms with special consideration for the necessary universals propositions and for the syllogism of necessity. The relationships between subject and predicate of the universal propositions and the consequences of these relationships for the particular propositions will be given particular attention to.

I shall concentrate my attention on Aristotle' differentiation between entites being individual (or: numerically one), on the one side, and entities being universal (or: not numerically one), on the other side: Aristotle distinguishes in many passages of his works (for instance, in Categories 5 and in Metaphysics Zeta 8 and 13) entities being a "this something" or a "this such" from entities being a "such". "These somethings / These suches", on the one side, and "Suches", on the other side, constitute realms of reality which are not to be confused with each other: These kinds of entites belong to realms of reality which are mutually incompatible. Individual entities are instances of properties; universals are (or; represent) properties which are instantiated by individual entities.

The field of the instances is always constituted by individuals, while the whole field of existence is constituted by individuals and universals (or: by instances and by properties as

potentialities for being-concretized in the instances). The way of existence of universals is different from the way of existence of the instances; individuals and universals exists at different levels, they live, so to speak, on different ontological types. I shall furthermore defend the thesis that universals – at least the universals corresponding to biological properties such as “being an animal” or “being a man”, do make part of Aristotle’s ontological universe (i.e.: Aristotle is not a nominalist).

Moreover, in order to explain the consequences deriving from a confusion between realms of reality, I shall take into consideration Aristotle’ lost work “De Ideis” with particular attention to the Argument of the One Over Many and to the Third Man Arguments; a few words will be dedicated to pointing out that Aristotle’s universals accomplish a completely different ontological duty from Plato’s ideas: this part of my exposition will aim at showing that, even if Plato’s ideas and Aristotle’s universals were completely identical in their features, notwithstanding the difference as to the ontological duties of both entities would make them completely different entities from each other. I shall finally defend Aristotle’s interpretation of substance and Aristotle’s conception of ontology against tropes ontologies (for instance, against Keith Campbell’s interpretation of ontology), and I shall describe Ernest Jonathan Lowe’s Four-Category-Ontology as – in my opinion – the most convincing interpretation of Aristotle’s ontological aims.

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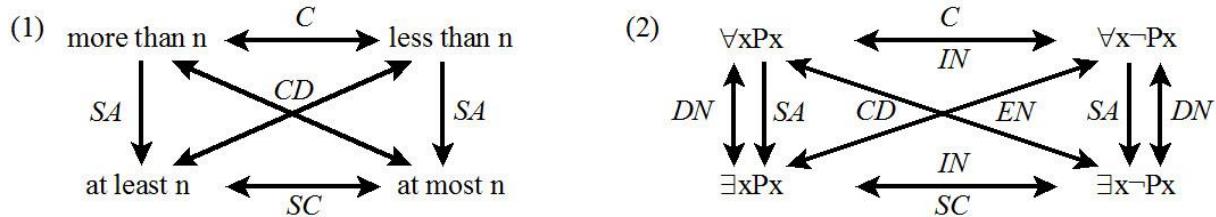
Aristotelian and Duality Relations with Proportional Quantifiers

Hans Smessaert

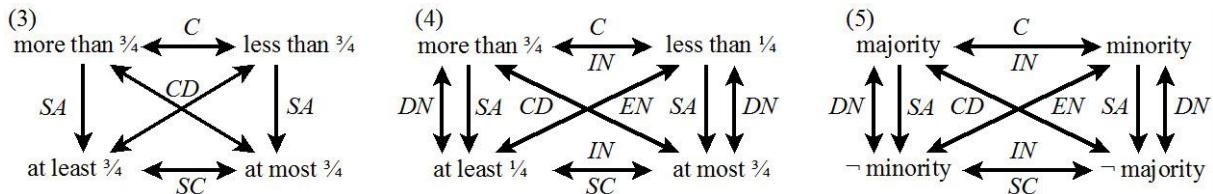
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The Aristotelian relations of contradiction (CD), contrariety (C), subcontrariety (SC) and subalternation (SA) have been argued to be conceptually independent of the duality relations of internal negation (IN), external negation (EN) and dual negation (DN) [1, 2, 4]. For any fragment of 4 formulas (from a logical language L for a logical system S) which is closed under negation – i.e. which consists of two pairs of contradictories -- the former set of relations can be diagrammatically represented as a (possibly degenerate) ARISTOTELIAN SQUARE, whereas the latter set gives rise to a (possibly degenerate) DUALITY SQUARE. Some such fragments only constitute an Aristotelian square -- as is the case for the numerical quantifiers in Figure 1 --, whereas others yield both an Aristotelian and a duality square simultaneously -- as is the case for the quantifiers of Standard Predicate Logic in Figure 2.



The set of Aristotelian relations is fundamentally hybrid: (i) CD, C and SC are symmetric and defined in terms of being true/false together, whereas SA is not symmetric and defined in terms of truth propagation [5]; and (ii) CD is a functional relation, but C, SC and SA are not. All duality relations, by contrast, are symmetric and functional. A further mismatch concerns the fact that the single duality relation of IN seems to correspond to two Aristotelian relations, viz. either C or SC. On a more abstract level, Aristotelian relations have been shown to be highly logic-sensitive, whereas duality relations are insensitive to the underlying logic [2, 5].



The central aim of the presentation is to chart which of the above logical relations hold between quantification formulas expressing the notion of proportionality. Two types of expressions will be distinguished: (i) EXPLICIT PROPORTIONALS such as *at least two thirds of the A's are B or less than 20 percent of the A's are B*, in which the proportion is explicitly referred to in terms of fractions or percentages; and (ii) IMPLICIT PROPORTIONALS such as *a minority/majority of the A's are B*, in which the actual proportion remains implicit. Explicit proportionals will be argued to give rise to (at least) two constellations: (i) the square in

Figure 3 corresponds to that in Figure 1 in being an Aristotelian square only, whereas (ii) the square in Figure 4 corresponds to that in Figure 2 in being both an Aristotelian and a duality square. Implicit proportionals, then, automatically yield ‘double’ squares, as in Figure 5. The analysis is carried out within the framework of Logical Geometry (www.logicalgeometry.org) and makes use of so-called bitstring representations, which are compact combinatorial representations of the denotations of the various types of proportional expressions that are based on (scalar) partitionings of logical space. Finally, since these proportional expressions are generalised quantifiers, their monotonicity properties will also be studied [3].

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Logical Rectangle

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The unicellular main-auxiliary algebra for single set theorems with P and Q two fact propositions is shown in Fig. 1.

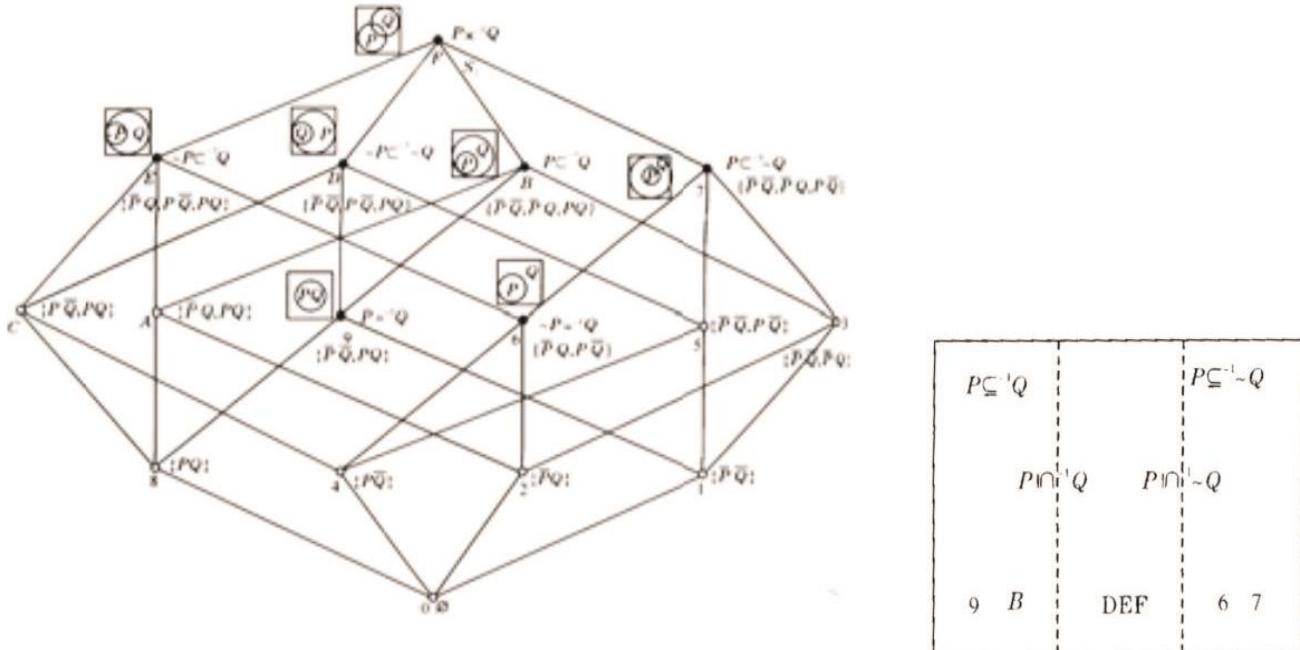


Fig. 1: The algebra

Fig. 2: Logical rectangle

In Fig. 1, the 7 concrete vertices are the 7 unicellular propositions of mutually-inversistic logic borrowed from Keynes' 7 models. Here, $P \subset^{-1} Q$ means that P is a sufficient but not necessary condition of Q . $P =^{-1} Q$ means that P is a sufficient and necessary condition of Q . $P \times^{-1} Q$ means that P intercrosses Q . $P \subseteq^{-1} Q$, a multi-cellular proposition, meaning P is a sufficient condition of Q or all P are Q , is denoted by the concrete vertices of the subgraph with B at the top and 0 at the bottom, that is, vertices 9 and B; the common feature of which is the absence of $P \cap \sim Q$. $P| \cap^{-1} Q$, a multi-cellular proposition, meaning some P are Q , is denoted by the concrete vertices of the subgraph with 8 at the bottom and F at the top, that is, vertices 9, B, D, E, F; the common feature of which is the presence of $P \cap Q$.

From Fig. 1, logical rectangle can be built, shown in Fig. 2.

In Fig. 2, $P \subseteq^{-1} Q$ is denoted by the left compartment, $P \subseteq^{-1} \sim Q$ is denoted by the right compartment, $P| \cap^{-1} Q$ is denoted by the left-middle compartments, $P| \cap^{-1} \sim Q$ is denoted by the middle-right compartments. Fig. 2 has all the 6 opposition relations the square of opposition has. For example, the subalternation of $P \subseteq^{-1} Q$ and $P \cap^{-1} Q$ is denoted by $P \subseteq^{-1} Q$ being contained in $P| \cap^{-1} Q$. The contradiction of $P \subseteq^{-1} Q$ and $P| \cap^{-1} \sim Q$ is denoted by the intersection of $P \subseteq^{-1} Q$ and $P| \cap^{-1} \sim Q$ being empty set, the union of $P \cap^{-1} Q$ and $P| \cap^{-1} \sim Q$ being universal set. The contrariety of $P \subseteq^{-1} Q$ and $P \subseteq^{-1} \sim Q$ is denoted by the intersection of $P \subseteq^{-1} \sim Q$ being empty set, the union of $P \subseteq^{-1} \sim Q$ not being universal set. The subcontrariety of $P| \cap^{-1} Q$ and $P| \cap^{-1} \sim Q$ is denoted by the union of $P| \cap^{-1} Q$ and $P| \cap^{-1} \sim Q$ being universal set, the intersection of $P| \cap^{-1} Q$ and $P| \cap^{-1} \sim Q$ not being empty set. Fig. 2 also reveals some information that the square of opposition does not have: the middle compartment is one that $P \subseteq^{-1} Q$ and $P \subseteq^{-1} \sim Q$ do not cover and that $P| \cap^{-1} Q$ and $P| \cap^{-1} \sim Q$ overlap.

Existential Import and Generalized Quantifiers

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In this talk we analyse the existential import carried by various generalized quantifiers and various higher order functions like those denoted by EACH-OTHER or THE-SAME. It is shown that, as in the case of the traditional square of opposition, the absence of the existential import associated with such functions preserves some nice logical properties of the square they form. For instance, the sentence Leo and Lea read the same book is true if no book has been read by anybody (in other words Leo and Lea read zero books and thus the "same books"). But in this case the sentence with the negated transitive verb is also true because the books that Leo and Lea did not read also the same (namely all books). We do not have this property if we suppose that Leo or Lea read at least one book.

Similarly, one can associate an existential import with exceptive quantifiers like Every student except Leo or No teacher except ten. In the first case the reading with the (generalized) existential import forces to recognize that there is one student more in

addition to Leo and without existential import we have to suppose that only one student (Leo) exists. For the second quantifiers, we have to suppose that there are eleven students (with the existential import) or respectively ten (without the existential import).

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