III WORLD CONGRESS ON THE SQUARE OF OPPOSITION

HandBook

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1. Third World Congress on the Square of Opposition

1.1. The Square : a Central Object for Thought

The square of opposition is a very famous theme related to Aristotelian logic dealing with the notions of opposition, negation, quantification and proposition. It has been continuously studied by people interested in logic, philosophy and Aristotle during two thousand years. Even Frege, one of the main founders of modern mathematical logic, uses it.

During the 20th century the interest for the square of opposition has been extended to many areas, cognitive science ultimately.

Some people have proposed to replace the square by a triangle, on the other hand the square has been generalized into more complex geometrical objects: hexagons, octagons and even multi-dimensional objects.

1.2. Aim of the Congress

This will be the 3rd world congress organized about the square of opposition after very successful previous events, in Montreux, Switzerland in 2007 and in Corté, Corsica in 2010.

The square will be considered in its various aspects. There will be talks by the best specialists of the square and this will be an interdisciplinary event gathering people from various fields: logic, philosophy, mathematics, psychology, linguistics, anthropology, semiotics. Visual and artistic representations of the square will also be presented. There will be a music show and movies illustrating the square.

The meeting will end by a final round square table where subalterned people will express their various contrarieties, subcontrarieties and contradictions.

1.3. Scientific Committee

- RENÉ GUITART, *Dpt of Mathematics, University of Paris 7, France*
- LARRY HORN, *Dpt of Linguistics, Yale, USA*
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1.4. Organizing Committee

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- FABIEN SCHANG, *Archives Poincaré, Nancy, France*

- JURGIS SKILTERS, *University of Latvia, Riga*

2. Plenary Lectures

Iranian Logicians on the Exceptions to the Existential Import

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Iranian logicians, from Farabi to contemporary scholars of both the seminary and the university, have made great contributions to the "square of opposition" in five important fields: 1) exhaustive study of the opposition between both the concepts and the propositions, 2) giving explicit detailed account of those oppositions about which no explicit explanation can be found in Aristotle's writings, i.e. subcontrariety and subalternation, and properly naming them, 3) closely scrutinizing the existential import (or the principle of presupposition) as the necessity of priority of subject's existence over predicate's existence for the subject, being used in affirmative categorical propositions in which the existence of the subject is necessary, 4) detailed inquiry into the exceptions of the existential import, and 5) comparing modern logic and classical logic in terms of the existential import, trying to resolve the disagreement between them in the foundations and implications of this principle.

Defining and clarifying the existential import as an axiom or self-evident principle or necessary rule (according to various perspectives of different philosophies), with its own ontological and epistemological explanations, the author concentrates on the inquiries into its exceptions. This principle has been applied by Iranian philosopher-logicians in several topics such as the truth of the affirmative propositions, mental existence, affirmedness of the possible non-existent, and nonaddition of existence to essence. Instead, its application to some other topics (e.g. predicating/attributing existence to essence) may bring about serious difficulties. Intellectual efforts of Iranian scholars at solving the related problems have formed an important chapter of the history of logic that will be critically reviewed in the paper.

Under the Sign of the Hexagon

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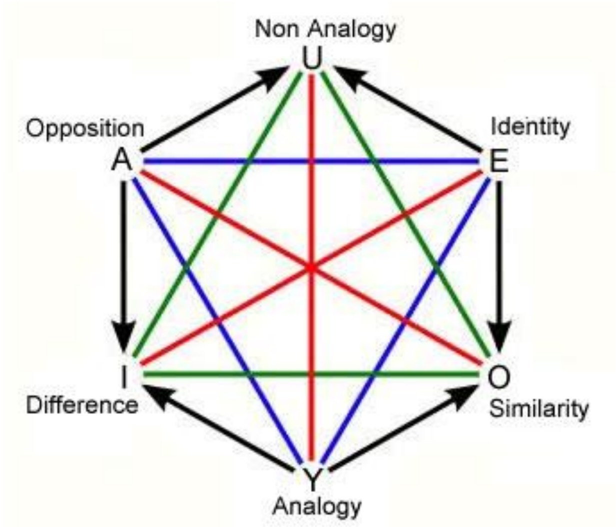
First I will present the general abstract theory of opposition underlying the hexagon of opposition, picturing the three squares of opposition that are within the hexagon and presenting a metahexagon of opposition.



Then I will show how the hexagon of opposition is useful to develop conceptual analysis in many different fields: from metalogic to traffic signs, through economy, music, semiotics, religion, poetic and opposition theory.

The Square of Opposition and Visual Cognition

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The focus of this paper is on visual images of the categorical propositions involved in the square of opposition. Although diagram has been used in logic and mathematics from antiquity it is only recently that system of diagrams is being considered as logic. The credit mainly goes to Sun Joo Shin whose Ph. D. dissertation re-opens the debate after Euler, Venn and Pierce. We have argued in some of our earlier papers that diagrams as the language of a logic have got an edge over strings if these (the diagrams) help in mental clarity, are rather simple and capable of expressing cognitive complexes visually. The square of opposition contains three opposites viz. contradiction, contrary and subcontrary. We shall examine the existing diagrams (squares, hexagons, octagons etc.) from the angle of representing the above oppositions visually.

We then present four possibilities of the relationship between an individual and two mutually exclusive concepts , present their visual representations and investigate into similarities with the square of opposition with respect to categorical propositions. A particular case of this arises when we take a concept and its negative concept. This study converges into an account of negation as absence of an individual in a

concept. “Absence” will be considered as a positive category as was the practice in ancient Indian discourses in logic. What would be the shape of opposition in terms of absence (instead of propositional negation) is an interesting query.

Visual representation of oppositions in presence of modalities shall also be touched upon. We shall see clear visuals of interior, exterior and boundary of a set and various negations generated thereby.

Avicenna on Possibility and Necessity

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In this paper I analyze Avicenna's theory on modalities and modal oppositions. My problem is: How does Avicenna define possibility, impossibility and necessity? What oppositional relations are there between modal propositions, whether quantified or not? To answer these questions, I will start by Avicenna's characterization of possibility which, according to him, has at least three meanings: the general possible, which is defined as 'not impossible', the 'narrow-possible' (as Wilfrid Hodges translates it) which is defined as 'not necessary' by Avicenna. But 'not necessary' means more precisely 'neither necessary nor impossible' since it is "that which neither its existence nor its non existence is necessary". It corresponds to what is now called the bilateral possible and is incompatible with both necessity and impossibility. The third meaning is a narrower sense of possibility and means: neither actual nor necessary. However this last meaning is not considered as adequate, and it is the second meaning that Avicenna retains although he does also use the first one. His classification of modal non quantified propositions contains then six levels of modalities in *Al ’Ibāra* instead of the four Aristotelian levels, because he includes the narrow-possible and its negation. This leads to the following results:

1- Avicenna gives the exact negation of the bilateral possible (considered as the real meaning of possibility), which is 'Necessary or Impossible', as has been shown much later by Blanché and others.

2- He applies the bilateral sense of the possible to quantified propositions and gives their exact negations. For instance, the exact negation of '**A** narrow-possible' is 'Necessary **O** or Necessary **A**'. He also gives in *Al Qiyās* the exact negation of the 'narrowest-possible' when applied to **A** and to **E**, but this kind of possibility is not included into the classification of consecutive modal propositions presented in *Al ’Ibāra*.

3- We find also a distinction between the scopes of modality in possible sentences since Avicenna separates between the following two sentences: "Every man is possibly a writer" and "It is possible that every man is a writer". The truth conditions of these sentences are different because the first one is true while the second is not obviously true. This could be seen as an anticipation of the medieval distinction between *de dicto* and *de re* readings of possibility.

The figures that emerge from this treatment of modalities seem to be the following:

1- A hexagon of modal singular propositions, since Avicenna includes the narrow-possible in his classification of the consecutive modal propositions. But this hexagon does not contain the subcontrariety relations.

2- Two more hexagons involving the usual modal quantified propositions plus the narrow-possible **A** and **E** and their negations. The modalities used in these quantified propositions are *de dicto*, i.e. external.

However Avicenna's treatment of the modal oppositions between quantified propositions remains incomplete because he does not consider all the relations that may occur between them. Furthermore, although he distinguishes between *de re* and *de dicto* formulations of quantified modal propositions he does not examine precisely the way

Contrariety in Aristotle's theory of being in *Metaphysics* 10

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For Aristotle in the *Metaphysics*, contrariety is not primarily a way to relate pairs of propositions, but describes rather relationships among beings. In *Metaphysics* Book 10, Aristotle says that all members of a *genos* are to be derived from two particular members which he calls the contraries of the *genos* (cf. 1055a8-9, 55b16-17). One contrary is the "one" or "measure" of the *genos*. Its privative contrary is the most deficient member. For example, in colours, the one is white; its contrary is black; and all other colours are composed of them. In *gene* of living things, characteristics of contraries are more complicated. In a genos of animals, the one will be the member with the maximal capacities, the privative contrary the one with the least. Since privation is often a contradiction (*antiphasis*) or incapacity determined in a recipient (10.1055b3-8), contraries can possess contradictory states. Other members of a *genos* Aristotle calls intermediates (cf. 1057a19-26), which he says are all composed from the contraries through privation, by degrees, of the capacities of the one (cf. 1057a18-19, 57b2-4, b30, 1055a8-9, 1055b11-15; 1005a3-4). So, Aristotle relates all beings within a *genos*, and all *gene*, via his notion of privation.

Aristotle says it is impossible for there to be intermediates from things that are not opposed (1057a32). At first glance, this seems to conflict with

Cat. 5.3b24-7, which appears to rule out substances being contraries in this sense. But “contrary” is homonymous (cf. *Met*. Δ.10), for contraries within a *genos* are not absolute contraries, for they share the common characteristics of the *genos*, e.g., all footed *vivipara* have feet and the females give birth to live offspring. They are, rather, *relative* contraries. If contraries are relative to a *genos*, and have intermediates, it would seem that contrariety is not dichotomous. Moreover, if contraries can possess contradictory states (*vide supra*), but, at the same time, have intermediates, then in some *gene* there seem to be intermediate states between contradictory states.

Epistemological Reflections on the Fregean Square of Opposition

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The distinction between "judgeable content" and "judgment" can be considered the most important result of Frege's epistemology. This idea is at the center of our discussion that concerns three fundamental theoretical points.

First, the distinction between "function", namely the fixed part of an expression and "argument", namely the variable part of it, plays the fundamental epistemological role to indicate when the argument is "determinate" or "indeterminate". This very distinction is relevant for specifying a new notation of "generality", which differs from the Aristotelian one and rests on a substitutional strategy. We'll consider Frege's replacement of the syllogism with a formal structure divided into "function" and "argument" by referring to his original notation in the *Begriffschrift*.

Second, the change of the Aristotelian notation gives rise to a criticism of the aggregative strategy presented by Boole. Judgment is not formed by aggregation out of previously given constitutive concepts. Frege's notation admits differences of concept's level that overcome the classical distinction between subject and predicate. The logical form, composed of "unsaturated" and "saturated" parts could be considered by itself as predication; consequently we do not need to some additional factor to relate them predicationally. What remains unclear in syllogistic is the crucial problem of “concept formation” as it starts from the assumption that concepts are formed by abstraction from individual things and that judgments express comparison of concepts like inferences. According to Frege, Boole’s logic presents the same problem.

Third, we point on the role of the difference between judgment and judgeable content in different kinds of speech acts. We must concentrate on the role of the sign for assertion that clearly shows the distinction between assertion and predication namely to combine a subject with a predicate does not imply that one performs an assertion about what the subject named. As we can notice also in the *Begriffschrift* Frege wants to avoid the inclusion of the assertion into the predication as it is incoherent. So, the task of the sign of assertion is to overcome this incoherency and to clarify that the negation and the distinction between universality and particularity do not apply to the judgment or the assertion but to a possible judgeable content.

To What Extent do Knowledge Models Exploit the Square of Opposition or its Extensions ?

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The square of opposition is exploited in computer science, particularly in engineering knowledge-based systems, since its very beginnings, which is rooted in the works of modern logicians of the late 19th and early 20th centuries, predominantly Frege and Gödel. Frege's results led to propositional logic and predicate/ first-order logic. Gödel's results enabled representing axiomatic systems algorithmically, which in turn enabled Turing to specify the first theoretical (mathematical) computer, the Turing Machine in 1936. Zadeh's first contribution to fuzzy logic in 1965 provided a means for representing any possible human logic. It has facilitated modelling possibilistic systems in all science and engineering disciplines, including fuzzified extensions of the square of opposition. Cognitive models for artificial intelligence make extensive use of these concepts, with the objective to mimic smart behaviours.

Engineering a smart behaviour requires modeling a particular logic, such that humans may perceive it as satisfiable and consistent, i.e. meaningful. The square of opposition is one logical concept that can help designing a meaningful system behaviour. We will explore to what extent such concepts were common in engineering knowledge.

Logical Pluralism and the Square of Opposition in Formal Ontology

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Formal ontology, contrary to philosophical ontology, is a field which is highly multidisciplinary comprising logicians, computer scientists, semioticians, linguists, cognitive scientists, and philosophers. This brings heterogeneity, both in methods and application areas, with an extensive scope ranging from general conceptual modelling to architecture and engineering, from the life sciences to the arts etc. It is therefore not surprising that logical pluralism plays an important role in the engineering of formal ontologies, with logics being used ranging from classical propositional and first-order logic, to intuitionistic, modal, description, and non-monotonic and paraconsistent logics.

This talk takes up the challenge of finding structure in this complex landscape by using the square of opposition and related logical shapes as a guide. We distinguish three levels, the micro level (structure within an ontology), the meso level (structure between ontologies), and the meta level (structure about an ontology). Specifically, we are interested in investigating the interplay between the triangle, the square and the hexagon, and logical pluralism, given that the modules of a heterogeneous ontology may employ different notions of local negation.

Some Post-syllogistic Structures of Opposition

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There is no doubt that the square of opposition has known a “new rising” in recent years, thanks to several scientific projects and events, carried out by scholars that meet regularly, most of whom do participate to this world congress (either present or past editions). A look at the scientific program of this congress shows the richness of the questions raised and the disciplines involved. It is tempting to say, but not to regret, that this variety makes it more difficult to say what the square really is and what it is for. The aim of this talk is to share few thoughts inspired by several not-well-known post-syllogistic structures of opposition, designed mostly by nineteenth-century logicians. This discussion will be explored in three main directions. First, we will consider the square as a visual device. As such, it invites and deserves a semeiotic analysis that might reveal interesting features that are so basic at first look that one might easily (but not happily) overlook them. Second, we will consider the place of the square within the history of logic. Indeed, it would be unfair to complain about the neglect of the square since the nineteenth-century if the square didn’t help with the new problems that faced the logician at the time. A quick look at such problems and what benefit there would be in using the square might give some indication on its logical status. Finally, we will look at some developments beyond the square and the use of more complex structures by past and present logicians, and the increasing interest in the mathematical properties of those structures.

The Hexagon of Opposition in Music

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In what ways can the hexagon of opposition formalise what the musician calls "musical logic" and by doing so aid him in better understanding what "negation" and "deduction" mean for the music at work?

To respond to this question, this presentation will propose successively three ways of appropriating this hexagon for musical realities: the first – briefly outlined – will depart from the diatonic/chromatic opposition; the next – sketched with more detail – will touch upon the rhythm/timbre opposition. Finally, the third – largely developed – will undertake to formalise properly musical discursivity by orienting it according to three principles. These in turn will be counterposed term by term to the great logical principles of Aristotle: a principle of *ambiguity* (as opposed to the principle of *identity*), a principle of *constrained negation* (as opposed to the principle of *non-contradiction*) and a principle of the *required middle* (as opposed to the *excluded middle*).

These three principles configure the musical composition as an interaction between three "entities" (an initial entity, its alteration and an entity of another type), an interaction that this presentation will undertake to formalise in the form of a hexagon of opposition. The detailed interpretation of this will suggest in return that the development of musical discourse (that which takes the place of musical "deduction") operates as a

Borromean knotting of three forms of alteration (musically taking the place of “negation”).

Contingency Logics and Modal Squares of Opposition

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As is well-known, in the tradition of logical studies one can find two notions of contingency, a weak and a strong one - the former being coincident with non-necessity, the latter being coincident with bilateral possibility (what can be true and can be false).

While all modal notions belonging to the Aristotelian square may be defined in terms of contingency in the weak sense, it is usually held that the strong (proper) notion of contingency may be used to define necessity only if necessity satisfies the property T (i.e. $\Box p \supset p$) but is uneffective to define weaker notions of necessity.

In the first part of the paper it will be shown, however, that extending contingential language with at least a suitably axiomatized propositional constant a contingential definition of necessity may be found even for weak notions of necessity.

In the second part of the paper the relations between contingency and necessity will be visually represented by squares of opposition. In this connection two basic notions are introduced: the one of a degenerate square of opposition and the one of composition of squares. Some elementary properties of such notions will be proved: e.g. that the composition of distinct squares of oppositions yields a non-degenerate square of opposition.

It will then be shown that contingential notions may be represented via degenerate and non-degenerate squares of oppositions. As a final step, it will be shown how the standard square of modal notions turns out to be equivalent to a suitable combination of contingential squares.

A Formal Structure of Global Debates by n-Opposition Logics

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One may consider global debates as localized within the ideosphere. Stakeholders of the ideosphere exploit rational tools of "theory construction" within important domains such as science, politics, art, religion. The systems produced by those stakeholders are a representation of both natural phenomena and human creations; moreover they embody scope and logics internally. Externally a rational debate among stakeholders should exhibit differences, constraints, contradictions and paradoxes only under the assumption that critics on systems may be submitted to global logical criteria.

In this talk, we present a new way to understand the relations between global debates and shared mental constructs (systems) by exploiting geometrical structures – logical squares, hexagrams, cubes, hypercubes – existing in a 6-opposition logic.

Towards a Calculus of Squares

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The deductive relations between the four corners of the square suggest a system with primary (strong) and secondary (weak) assertion and denial as four forms of judgement. This can be well understood by using ideas from extended logic programming and definitional reasoning. Proof-theoretically it can be captured by extending Gentzen's calculus of sequents to a calculus of squares, which has four rather than two entries for formulas.

3. Abstracts of Contributors

Square of Oppositions and Hexagons in Argumentation

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The idea of opposition plays an important role in argumentation. Apothéloz has pointed out the existence of four basic argumentative forms, where two negations are at work: i) "*y* is a reason for concluding *x*" (denoted $\mathtt{C}(x) : \mathtt{R}(y)$), ii) "*y* is not a reason for concluding *x*" ($\mathtt{C}(x) : -\mathtt{R}(y)$), iii) "*y* is a reason against concluding *x*" ($-\mathtt{C}(x) : \mathtt{R}(y)$), and iv) "*y* is not a reason against concluding *x*" ($-\mathtt{C}(x) : -\mathtt{R}(y)$). These four statements can be organized in a square of opposition (modifying a recent proposal by Salavastru where the vertical entailments were put in the wrong way). Indeed, if *y* is a reason for not concluding *x*, then certainly *y* is not a reason for concluding *x*. It leads to the argumentative square of opposition:

$$\begin{tikzcd} \mathbf{A}: C(x) : R(y) \arrow[r, dash, dashed] \arrow[d] & \mathbf{E}: -C(x) : R(y) \arrow[d] \\ \mathbf{I}: -C(x) : -R(y) \arrow[r, dash, dotted] & \mathbf{O}: C(x) : -R(y) \end{tikzcd}$$

Moreover, it is possible to build a hexagon of opposition (in Blanché' sense) by considering the different possible consequence relations linking a reason *y* to a conclusion *x*; see below where we use the traditional naming of vertices, $|\vdash$ denotes entailment and $||\!/\!-$ its negation):

U: $y \mid\vdash x$ or $y \mid\vdash \neg x$; **A**: $y \mid\vdash x$; **E**: $y \mid\vdash \neg x$; **I**: $y \,\|/\!- \neg x$; **O**: $y \,\|/\!- x$; **Y**: $y \,\|/\!- x$ and $y \,\|/\!- \neg x$.
Besides, the link between a conclusion and a reason may be strong ($\vdash$) or potentially defeasable ($|\sim$). This leads to the following hexagon showing the interplay between a strong consequence relation ($\vdash$, and its negation $|/\!-$) and a weak consequence relation ($|\sim$, and its negation $|/\!\sim$), where having $y \vdash x$ and $y \,|/\!\sim x$ is supposed to be impossible: **U**: $y \,|/\!\sim x$ or $y \vdash x$; **A**: $y \vdash x$; **E**: $y \,|/\!\sim x$; **I**: $y \mid\sim x$; **O**: $y \,|/\!- x$; **Y**: $y \mid\sim x$ and $y \,|/\!- x$.
Note that **A**, **E**, **Y** refer to 3 mutually exclusive situations in both cases.

Higherdimensional Oppositions from Proof-Theory

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We discuss a refinement of the general setup of opposition shapes for a given logic: Usually one considers semantic relations between formulas, for example that not both can be true. For a complete logic this relation can be expressed as saying that there exists a proof with the two formulas as hypotheses which arrives at a contradiction. Now instead of letting the edges in an opposition diagram indicate the mere existence of such a proof one can let every edge denote a particular such proof. We will show how this results in a natural interpretation of the triangles enclosed by the edges: They can be interpreted as the statement that the proof denoted by one edge is equal to (or transformable into) the composition of the proofs denoted by the remaining edges. We also discuss how opposition shapes are transported along translations of logics and when and how the possibility of creating opposition shapes can be seen as an algebraic structure on a logic.

A Square of Oppositions Arising from the Comparison of Gentzen's Natural Deduction and Sequent Calculus

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The correspondence of Gentzen's natural deduction and sequent calculus is based on the idea that formulas in the antecedent and succedent of sequents correspond, respectively, to assumptions and conclusions of natural deductions. In this paper, we are interested in how exactly this correspondence should be understood when either the antecedent or the succedent of sequents are empty.

Since a natural deduction tree has all nodes labelled with formulas, there is no genuine notion of 'no assumptions' or of 'no conclusions' immediately available in this framework. Typically, sequents with an empty antecedent correspond to deductions in which all assumptions are discharged; and sequents with an empty succedent to deduction having the unit "Falsum" as conclusion.

The reason of this asymmetry is investigated, by considering the case of dual-intuitionistic logic, in which discharge applies to conclusions and not to assumptions, and in which we have the unit "Verum" instead of "Falsum".

By correlating the two frameworks, the two units and the operations of discharge of assumptions and conclusions yield four possible 'forms of judgement' which, it has been suggested, may be viewed as forming a square of opposition.

To make the idea precise, we consider the sequent calculus for linear logic, in order to (i) establish a correspondence between the operations of assumptions and conclusions discharge on the one hand and linear logic multiplicative units on the other; and (ii) between Verum and Falsum on the one hand and linear logic additive units on the other.

The Logic of Paradoxes: Its Root in Legal Practices and its Evolutionary Multi-valued Character

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The logic of uncertainty and paradoxes has its origins in the geometric figure of the square, the Grecian tetragram or rather the Sanskritic catuskoti. Its ancient kernel is the old Indian, Grecian resp. biblical thinking and concept of justice.

This talk tries to reveal that:

a) This view of legal practice, ontological and logical classification presupposes human beings' free will.

b) The principle of square overrules any sequential logic, but following its own inherent structure GÖDEL'S liar induces a dynamical multi-valued logic which will be presented.

c) The question of consciousness, reality and the dichotomy of subject and object as well as of free will are closely related to this subject.

d) Based upon the technical formulation of b) HEGEL and NAGARJUNA have mirrorinverted position.

e) There occur some open questions concerning freedom, consciousness and the absolute well-ordering.

f) We will analysis its extension to the hexagon under the viewpoint of dynamic and mysticis

Antilogic, Counterlogic and the Square of Opposition

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We define an antilogic and a counterlogic for any logic. We proceed to study oppositional relations between a logic, its antilogic and counterlogic to show under which conditions they are display structures akin to the square of opposition. The work is part of a greater effort to explore the intersections of universal and modal logic towards a general conception of what a logic is and a revision of its role within research in metaphysics.

L’image, le signe, le réel : unités et consistances de l’objet poétique chez Aristote

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De quoi l’objet fictionnel et vérace de la poétique tient-il sa consistance, et par où peuvent donc s’indiquer à travers lui quelques facettes du réél, si cet objet ne copie rien de tel qu’une chose entièrement déjà préexistante et complètement achevée qui serait comme son modèle – l’Homme, par exemple, ou même *tel* homme, qui ne serait *qu*’Œdipe ; et surtout si cette consistance se joue à l’articulation complexe de l’image (du *portrait*), de la figure absente (du symbole) et du signe (orchestré en *logos)*? Qu’en est-il de cet universel (l’humain ? une *vie* ?) donné au travers d’une aventure si singulière (les décisions et actions d’une histoire particulière propre), qui n’est dès lors ni jamais l’*exemplaire* d’un ensemble unifié (la logique même de l’*ethos* implique que ce soit en étant pris dans les rets particuliers d’un être devenu, des habitudes, et désir et pensées qui font le caractère que l’on agisse, autant que l’inverse) ni non plus une *exception* opposable à l’uniformité supposée d’une classe (si Œdipe est un bon personnage, c’est qu’il éclaire sur ce que *peut* l’homme, quel qu’il soit) : le héros n’en est un, n’est une *figure* possible, signifiante et symbolique, qu’à la condition de faire à la fois saisir en lui une proposition universelle affirmative et de s’en donner comme l’obstacle interdisant de se suffire de la première. Œdipe a agi ainsi, donnant forme à un genre d’homme, mais dont *pas tous* par définition ne pourraient s'y rassembler simplement. Il faut, pour ce que la poétique révèle du réel humain, aller s'installer là où seules des propositions particulières affirmatives peuvent pourtant soutenir un universel *absent,* qui n'a pas d'unité en tant que tel ; *quelques* hommes agissent de telle ou telle manière, que seule *une* histoire, un parcours, une décision et un « nœud » narratif peuvent raconter en sa forme singulière, mais de ce « quelques » on ne peut pas tirer nécessairement qu'ils sont une instanciation (particulière) d'une universelle affirmative. Le héros (tragique, en l'occurrence) n'est jamais seulement non plus un élément d'un partitif indéfini – le problème sur ce point n'est pas tant le *nombre indéfini* des éléments comptant dans un hypothétique ensemble (« les hommes vertueux mais qui commettent une faute » par exemple), que le mode même d'unité de ce particulier, qui a vocation à donner l'ampleur suffisante pour conceptualiser, mais sans nommer jamais un universel.

On voudrait pointer ici plusieurs modes d'unité dont l'articulation ou le croisement serait en grande partie ce qui ferait tenir la chose poétique, et par là même s'indiquer quelque chose du réel humain : non pas en faisant le portrait d'un individu, unité simple et substantielle de l'actant, mais bien en jouant les unes avec/contre les autres une unité imaginaire, comme en un miroir, qui donne la forme idéale d'un type d'agir, un « schème général » ou unité narrative qui se donne en une unité divisible mais censé délivrer une unité symbolique (le sens des actes, des décisions, etc.), et une unité partielle, incomplète par essence, qui serait précisément celle qui n'existe nulle part, n'étant ni le corps matériel du personnage, ni la « totalité » englobante de « l'histoire » (avec la somme de ses nouements et dénouements qui articulent son mouvement comme un vivant). Ce qu'il s'agit de « comprendre » grâce à/ dans une bonne histoire n'est donc ni seulement l'attribution à *une* individualité sujet unique, la référence à une singularité comme cause suffisante, ni non plus l'exemplaire illustration d'une classe aux contours indéfinis. L'enjeu n'est précisément pas de quantification, mais de mode de consistance : cette unité de l'*acte*, de la décision et de la manière d'agir, qui signifie un « caractère » ou une personne, est proprement celle à la fois que le personnage lui-même *manque* sans cesse, sauf à l'entrevoir, et à retardement sans doute, *après* l'acte, et celle que le lecteur ne cesse de fabriquer, en tricotant ces fils de nature différentes et de vitesses différentes que sont l'unité narrative englobante, l'unité « imaginaire » donnée dans le « portrait » écrit, et l'unité symbolique, en mouvement.

En cela ce que la poétique donne à voir, même chez Aristote, est bien l'aventure d'un sujet « partiel » à lui-même, et dont rien, sinon d'être pris dans un récit dont il est à peine l'auteur, ne lui donne « l'unité » : seule la mise en récit, et la valeur fugitive de *signature* que prend l'acte, permet de faire exister cette singularité porteuse d'universalité.

The Medieval Octagon of Opposition for Sentences with Quantified Predicates

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The traditional square of opposition consists of four sentence types. Two are universal and two particular; two are affirmative and two negative. Examples, where "S" and "P" designate the subject and the predicate, are: "every S is P", "no S is P", "some S is P" and "some S is not P". Taking the usual sentences of the square of opposition, quantifying over their predicates exhibits non-standard sentence forms. These sentences may be combined into non-standard Squares of Opposition (an Octagon in this case), and they reveal a new relationship not found in the usual Square. Medieval logicians termed "*disparatae*" sentences like "every S is some P" and "some S is every P", which are neither subaltern nor contrary, neither contradictory nor subcontrary. Walter Redmond has designed a special language L to express the logical form of these sentences in a precise way. I will use this language to show how Squares of Opposition, standard and non-standard, form a complex network of relations which bring to light the subtleties contained in this traditional doctrine.

Groups, not Squares: Exorcizing a Fetish

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The idea of duality is but a mere simplification of the deep relationship between algebra and logic known as "quaternality" which lies behind De Morgan laws and behind many results of universal Boolean algebra, geometry, topology, and several other areas. Such quaternality relation has a rich structure, represented by the Klein 4-group, and from this perspective can be studied in much more general terms. I argue that the celebrated Square of Opposition is just a shadow of a much deeper relationship on duality, complementarity, opposition and quaternality expressed by algebraic means, and that any serious attempt to make sense of squares and cubes of opposition must take into account the theory of symmetry groups

Square of Expressions

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This paper aims at producing a double of the square of oppositions, the square of expressions:

1. By following Deleuze analysis in *Difference and Repetition*, I will first distinguish between two logics operating in Aristotle: the logic of the univocal concepts – as developed in the *Organon*, and the logic of the categories – as developed in the *Metaphysics.* As Deleuze puts it, these two logics reflect one another in so far as difference is submitted to identity, making possible the oppositional relation in the *Organon*, and the analogical relation in the *Metaphysics.*
2. The second part will be a historical survey of the effects of '*being'* on the square of expressions. In this investigation I will show how the introduction of *being* in the square of opposition brings upon the square confusion (Abelard), an explanation (Hegel) or acts as a condition of possibility (Shelling).
3. The final part will aim at showing how both the square of oppositions and effects of being on the square, understand being as an inclusive relation between the part and the whole which leads to the oppositional relation and syllogism. I will then propose a different interpretation of *being*, based on Deleuze' *Cinema 1&2*, to establish an *expressive* relation between the part and the whole. The last step will be the formalization of these relations in a *square of expressions*.

"Yes, Timothy, there really are valid logical squares"

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A well-known reason why the theory of opposition suddenly failed is its being invalid once subject terms are empty names: the logical square is said to be valid only if the domain of reference is non-empty, thus discarding its universal application. The present talk wants to argue that this historical fact should be corrected beyond any philosophical or linguistic assumptions.

Despite Strawson's attempt to restore its validity in 1952, Timothy Smiley launched in 1967 what seemed be a decisive charge against the logical square. Yet here is presented a way to validate the square by an alternative formalization of the quantified propositions, irrespective of ontological preconditions and with the sole framework of first order logic.

This results in a new set of eight propositions and an enriched logical cube of quantified propositions, including five valid squares and six valid hexagons.

Rough Sets and the Square of Opposition

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Rough set theory deals with incomplete information where the available attributes cannot describe reality accurately. More precisely, when the extension of a set of objects cannot be defined, it is approximated through a pair of sets, the lower and upper approximations, which are defined on the basis of an equivalence relation. In the original setting, these two approximations give rise to a tri-partition of a universe in objects which surely belong to the set, surely do not belong to the set and objects that cannot be correctly classified using the available knowledge. Thus, they generate a hexagon of opposition, which extends the usual square of opposition. In more general models, when the two approximations are not dual (that is, we cannot define one from the other), they give rise to a cube of oppositions. Finally, on top of an equivalence relation, we can consider its non-transitive generalization, i.e., a symmetric reflexive relation, and their opposites: discernibility and preclusivity relations, respectively. Then, these four relations can be represented through a classical square of opposition.

Heraklitean Oppositions

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The Greek philosopher Herakleitos (5-th century B.C.) has been attributed a theory of oppositions. Despite the sparsity of his legacy, his philosophy has been characterized by a great many commentaries as focused on paradoxes (epistemologically speaking), and on nature as in permanent flux (metaphysically speaking). A coherent interpretation of his work results if it is centered around a basically phenomenological (both metaphysical and epistemological) principle of opposition, or ‘distinction', as I prefer to call it. In this contribution I want to show how aristotelian oppositions can be derived upon this understanding of Herakleitos' remnants, in a logically and philosophically better motivated and more general way.

Between the Finite and the Infinite: What Kind of Negation?

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Our talk proposes an exploration of the logical relation of negation between the finite and the infinite through the square of opposition. We study the main conceptions of the infinite in mathematics and we propose to position them according to their logical relation of dependency with the finite. For instance, the Aristotelian infinite can be understood in negative terms as far as the quantity is concerned: the infinite is the subaltern negation of the finite. On the contrary, in the modern set-theoretical conception of infinite, the contradiction between infinite and finite puts the positivity on the infinite; the finite appears as the logical negation of infinite. The aim of this semantic presentation of the infinite is to understand the consequences of these two conceptions as far as knowledge and understanding are concerned. We'll analyze the pictorial representations of the mathematical infinite: how can geometricians and mathematicians overcome the obstacle of the pictorial construction of the infinite? We'll expose a semiotic analysis of the dialectical relation between pictorial and algebraic representation of the infinite that allows us to discern the cognitive stakes of these different conceptions.

The Square of Ontological Questions: a Flexible Erotetic Cure for *Mathematosis*?

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The starting point is a list of subsequent classical questions: (1) What is there? [*the classic completely generic metaphysical question*]; (2) Of what does reality consist?; (3) What is it to be an actual entity? [*the fundamental metaphysical question*]; (4) Do times exist?; (6) Is there at least one thing?; (7) How to live? [*the central philosophical question*]. But the metaontological question, namely – the question: ‘Do ontological questions really have answers?’ is introduced and briefly discussed in the context of meta-ontological monism and pluralism. Obviously, we attempt to avoid the camp of quizzicalists (in the Stephen Yablo’s sense). The phenomenon or rather the syndrome (i.e. a complex of symptoms that collectively characterize some abnormal condition) of the (Quinean) *mathematosis* is shortly analyzed. The famous hexagon of opposition associated with provability logic is adopted. Following Belnap an erotetic hexagon is developed by means of standard model-theoretic semantics for questions. The so-called non-reductionistic approach to questions is presupposed.

Mereological Hexagons for the so-called Paradox of Epimenides

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Epimenides says: 'Cretans always lie'. If this is true, then Epimenides always lies, since he is a Cretan. But if he lies, then the sentence: 'Cretans always lie' is false. Since this is false, there must be a Cretan who does not always lie. If we do not see ourselves justified to assume that there is a Cretan who does not always lie, then what Epimenides says, comes to the same thing as the liar paradox. But if we do assume that there is a Cretan who does not always lie, with the only excuse that an infinite regress is absurd and has to be stopped, then we happen to establish logically by *reductio ad absurdum* the existence of an empirical entity, a non-lying Cretan. Does this present a (non-semantic) paradox? Church ("The Liar by Alexandre Koyré", *The Journal of Symbolic Logic* 11 (1946), 131) and Prior ("Epimenides the Cretan", *The Journal of Symbolic Logic* 23 (1958), 261), and Kleene (*Introduction to Metamathematics*, Groningen: Wolters-Noordhoff, 1952, 39) said that it does. Koyré ("The Liar", *Philosophy and Phenomenological Research* 6 (1946), 348) was of the opposite opinion, but offered no arguments for this.

I use hexagons of opposition as heuristic devices on behalf of an argument which says that it is legitimate to assume *a priori* the existence of at least one non-lying Cretan: a "slice" of Epimenides himself. I.e. I provide support for Koyré's opinion *contra* Church, Prior and

Kleene. A temporal part of Epimenides which utters the truism ‘Cretans lie’, i.e. without the second word of the Epimenides paradox, forms a Cretan, i.e. a “slice” of Epimenides, who says something true. Another “slice” of Epimenides, the one which says ‘Cretans always lie’, is the lying Cretan. This makes Epimenides’s sentence: ‘Cretans always lie’ false, without any paradox emerging, either a semantic or a non-semantic one.

Logical Contingency in William of Sherwood's Modal Square of Opposition

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William of Sherwood's (1267 Paris manuscript) *Introductiones in logicam* offers for its time a useful standard manual of syllogistic term logic. In Chapter One on 'Statements', sections 21-28 on modalities, closing the chapter, and in particular in section 28 on 'The Interrelation of the Modes', William supplements his traditional Aristotelian square of opposition with a square of opposition for modal statements. It is logically isomorphic with the four modal categories Aristotle presents in *De Interpretatione*, yet William seems to propose diagramming the modal categories in a modal square of opposition that does not map directly onto the traditional AEIO categorical square of opposition. I am interested in the question of how William's modal square of opposition relates to Aristotle's and to consider some of the reasons why he may have transformed the positioning of modal categories in his square, according to his original mnemonic instructions. The question is graphic in the sense that it concerns which of possibly being, possibly not being, not possibly being, and necessarily being should be arranged to display their logical interrelations in two dimensions, to whatever extent they are exemplified, of contradictories, contraries, subcontraries, and subalterns, as in the categorical square. I focus on William's concept of what he calls logical contingency in one corner of his modal square, that seems to be his more strictly logical replacement for Aristotle's *De Interpretatione* reference to a normative epistemic concept of admissibility. I suggest that William makes the relatively common mistake of collapsing logical contingency into logically contingent *truth*, possible-*p* and possible-not-*p* versus p and possible-not-*p*, and that in this regard there is a more general cautionary lesson. I urge instead that logical contingency, properly conceived, is a matter of both the logical possibility of a proposition *and* of its negation. When this concept of logical contingency is considered, the failure of William's three forms in his modal square can be made formally explicit. I consider alternative interpretations of Williams' intentions in this part of the square to see if the difficulty is more charitably resolvable, and finally I investigate the implications of an appropriately revised understanding of the category of logical contingency in a correspondingly modified preferred graphic representation alike of Aristotle's and William's modal square.

Generalized asymmetry in the Natural Language Lexicon:
Quantifiers, Colour Terms and Number Words

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In previous work (Jaspers [2011] and Jaspers [to appear]), it was argued that relations of opposition between primary and secondary chromatic percepts can be shown to be algebraically the same as those between the three primary items (all, some, none) in the lexical class of quantifiers and their contradictories. This intriguing homology – set up using tools developed by Jacoby [1950], Sesmat [1951], Blanché [1953], Horn [1990] and Smessaert [2009] – manifests itself linguistically in parallel asymmetries of lexicalisation.

On the one hand, a subset of the logical and colour terms that are organised by the abovementioned system of oppositions is very commonly lexicalized cross-linguistically, as if by conceptual pressure (all, some; black, white, red). Other terms are less frequently lexicalized, with lexicalisation largely determined by utility or frequency (nand; magenta, cyan). Crucially, not only concepts in the O-corner (= bitstring 011 of the algebra) of the square of oppositions get no or only a non-natural single word lexicalisation (cf. Blanché [1969]; Horn [1972, 1989, 1990]), but the problem is more general. In predicate logic, for instance, it further affects the so-called U-corner (= 101 in the diagram below), as well as the level zero operator (000) and the universe (111). This is represented in the following figure, in which the corners that resist natural single word lexicalisation are boxed.

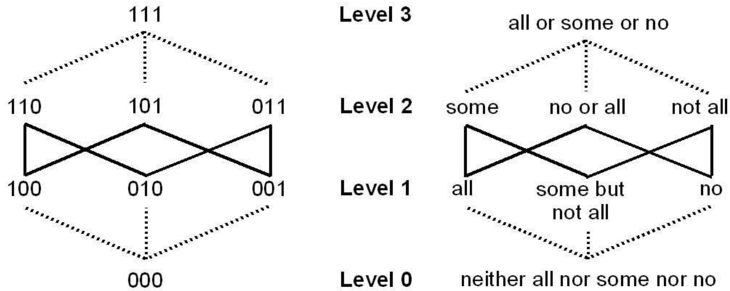

Figure 1: Hasse diagram for predicate logic

The pattern of asymmetry can be shown to extend to modality. Thus systems of alethic and deontic modality (the latter as analyzed by Von Wright [1951] for instance) have been identified as "systèmes dissymétriques" by Blanché [1969, pp. 93-94] and Horn [1972]. Blanché draws an incomplete hexagon without O (011) and U (101) to represent his version of von Wright's system, but it can easily be completed with two further boxed corners at Levels zero and three beyond the gaps noted already.

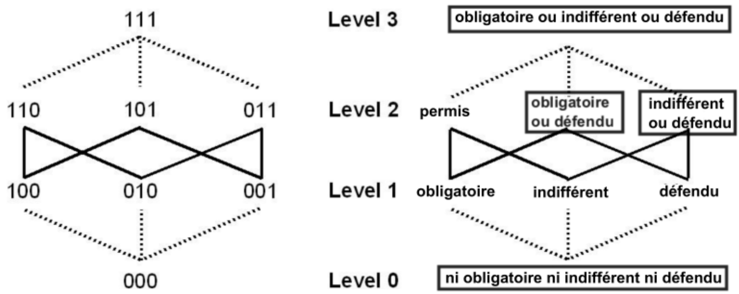

Figure 2: Hasse diagram for deontic logic (extending Blanché's version of von Wright (1951))

Our proposal diverges both from von Wright's and Blanché's analyses, however. As opposed to the former but in line with Horn [1989], we maintain that the notion indifferent should be replaced by bilateral permitted (but not obligatory) in 010 (Y-corner) and unilateral permitted (maybe obligatory) in 110 (I-corner), on a par with the occurrence of the item or in both the Y-corner (bilateral or exclusive) and the I-corner (unilateral or inclusive) of the propositional calculus. From Blanché we differ in that he postulates a fully natural occupant ("nullement une création artificielle") imperative (affirmatif ou négatif) for the U-corner, which he claims is the contradictory of indifférent. The objection here is not only that we are no longer seeking a contradictory of indifférent, given our revision of von Wright's 010. More importantly, the word impératif is really an A-corner word and the crucial addition (affirmatif ou négatif) to artificially turn it into a U-corner occupant could be added to obligatory no less than to imperative. Our main reason for boxing 101, however, is that a U-corner predicate expressing that the same deontic source obliges somebody to do something and simultaneously obliges him not to do that same thing is conceptually artificial and in view of that not naturally expressible by means of a simplex lexicalisation.

A final instantiation of the generalized asymmetry perspective to concept formation in closed lexical fields concerns natural numbers and the operations of addition and subtraction. The extension is partly suggested by the numerical properties of the atomic strings of the bitstring algebra, partly by asymmetries between odd and even numbers in terms of divisibility.

The resulting generalized bifurcation between natural and nonnatural lexicalisations and the binarity that characterizes concept formation in the realms of colours and numbers, suggests that Hauser et al. [2002]'s distinction between an internal computational system FLN (Faculty of language - narrow sense) and an internal computational system plus its accompanying “sensory-motor" and “conceptual-intentional" systems FLB (Faculty of language - broad sense) might be applicable not just to syntax, but also to concept formation and hence the lexicon.

The Hexadecagon of Opposition Applied to the Definition of ‘Knowledge’

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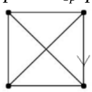
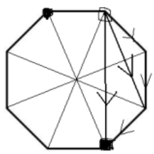
The definition of ‘knowledge’ evolves in Plato from belief; to true belief: $\sim B_{Sp} \bullet \sim p$ $\sim B_{Sp} \bullet p$

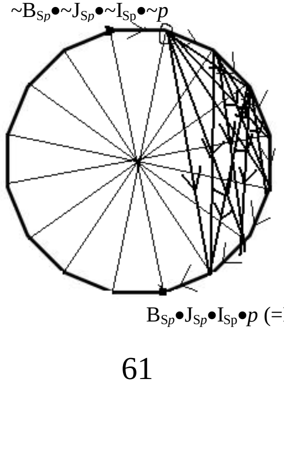


$B_{Sp} \bullet \sim p$ $B_{Sp} \bullet p$ (=K_{Sp}),
(belief revision depicted when *p* is true as S goes from not believing *p* to believing *p*); to justified true belief (JTB): $\sim B_{Sp} \bullet \sim J_{Sp} \bullet \sim p$

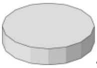


$B_{Sp} \bullet J_{Sp} \bullet p$(=$K_{Sp}$). (belief and justification revision depicted when *p* is true as S moves from combinations of justification and belief towards justified belief). After Gettier’s counterexamples in 1963, Lehrer added a fourth condition:

$\sim B_{Sp} \bullet \sim J_{Sp} \bullet \sim I_{Sp} \bullet \sim p$



$B_{Sp} \bullet J_{Sp} \bullet I_{Sp} \bullet p$ (=K_{Sp})

The arrows of belief, justification and indefeasibility revision lead to the final node of knowledge. The hexadecagon can be extended to a hexadecagonal prism . This would include possibilities of having indefeasible justified true belief of ~*p* as well as of *p*. Knowledge in general evolves from believing ~*p* to not believing *p* to believing *p*. I use the hexadecagon and the hexadecagonal prism to attempt to resolve Fitch's paradox of knowability.

Leibniz and the Square: A Deontic Logic for the *Vir Bonus.*

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17th century philosopher Gottfried Leibniz's contributions to metaphysics, mathematics, and logic are well known. Lesser known is Leibniz's "invention" of deontic logic, and that his invention derives from the Aristotelian square of opposition. In this paper, I show how Leibniz developed a "logic of duties," which designates actions as "possible, necessary, impossible, and omissible" for a "vir bonus" (good person). I show that for Leibniz, deontic logic can determine whether a given action, e.g., as permitted, is therefore obligatory or prohibited (impossible). Secondly, since the deontic modes are derived from 'what is possible, necessary, etc., for a good person, and that "right and obligation" are the "moral qualities" of a good person, we understand how deontic logic is derived from these moral qualities. Furthermore, on this deontic basis, Leibniz derives a definition of 'the good man' as 'one who loves everyone' (where 'love' is understood as practical).

Roland Barthes’ Semiotic Square of Opposition

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In my paper I analyze a fragment from Barthes' *Incidents*. I will focus my analysis of *Incidents* on relationship between the author, the narrator, the protagonist and the readers. Barthes does not offer a traditional narration but rather he creates an uncanny square of opposition between these figures in *Incidents* that are deceiving for readers. What strikes readers of *Incidents* is the style, which blurs distinctions between fiction and diary, fiction and criticism or theory, and between personal and public. *Incidents* is an inquiry into racism and homophobia but it does it not directly but via textual strategies of deconstructing the traditional square of relationship between the author, the reader, the narrator and the protagonist. I claim that via this strategy we can approach the text as radically open for reinterpretations. I claim that we can look at *Incidents* as an incitement to develop new narrations and more broadly new ways of thinking about sexuality and racial issues.

Some Remarks on Oppositions in Music

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In my talk I would like to analyze the evolution of some fundamental oppositions in modern European music theory, such as: silence/music (silence/sound, silence/noise, noise/sound), minor/major and consonance/dissonance etc. I will consider this question from the perspective of music theory, history of music and aesthetics.

Since Rameau and his "Traité de l'harmonie" (1722) musical compositions were based on the structure of oppositions that constituted the universal language of music. Development of musical forms was based on transformations of oppositions in many levels. Classical oppositions in music transformed into new contrasts or – in some cases – even disappeared. In my talk I will try to reconstruct historical and theoretical context of this evolution. I will also briefly consider if and how these musical oppositions can be organized.

Is Aristotle the Square of Opposition's Father?

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Starting from the question of the square's paternity, we want to examine the birth's context of the square. Facing « sophistic difficulties » (*logikas duskheiras*, in *Metaphysica* Gamma 3, 1005b22 ; compare with « *sophistikas enokhlêseis* », in *De interpretatione*, 6, 17a36-37), Aristotle shows that contradiction is possible if we conceive it as an opposition between an affirmation and a negation of a certain form – here is involved a specific theory of *logos*, that is to say a certain idea of the proposition as saying something of something, *legein ti kata tinos*, which is the subject matter of *De interpretatione*. This treatise offers the first logical definition of contradiction and more generally of opposition. But in *Categories* and *Topics*, Aristotle has an other idea of opposition, no longer logical, but semantic: he is referring to a fourfold classification of the meanings of « to be opposite ». We want to show how Aristotle is constructing his logical definition of opposition with the quantification of propositions and to compare precisely the square of opposition in Aristotle, Apuleius and Boethius, focusing on particular proposition. Finally, we shall see that even if Aristotle is the first to theorize logically opposition, his idea on negation is still floating, if we compare it to an other logic in antiquity, the stoic one. We shall conclude on the limits of the first square of opposition.

The Analysis of Truth in the *Port Royal Logic*: Extension and Distribution

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The *Port Royal Logic* has been judged a departure from the medieval logic in part because its analysis of truth in terms of extension is understood to be a rejection of the medieval theory of reference. Jean-Claude Parineté, for example, maintains that its analysis of truth in terms of *extension*, a new term in technical logic, is a sharp departure from medieval semantics because it makes a novel use "term restriction" that represents a rejection of the medieval theory of supposition and its explanation of term-reference through the logical consequences known as descent and ascent, which hold between quantified propositions and their particular instances.

In this paper I argue that these claims are greatly exaggerated. The concept of extension as Arnauld and Nicole use it, if not the term itself, was well understood in earlier logic. Moreover, contrary to Parineté's claim, its use by the *Logic* in the analysis truth is closely tied to the traditional theory of supposition and its technique of ascent and descent. There is novelty, but it lies elsewhere. What is new in the explanation of truth is its analysis in terms of what the authors call a *universal term*, which came to be known in later logic as a *distributive term* following the terminology more common in medieval logic. We shall see that: (1) prior to the *Logic* the term *universal*, aka *distributive*, term was defined within supposition theory and used in this sense to state the well-known set of rules that characterize the valid moods, the same rule set used in the *Logic* itself, and (2) the medieval concept in fact underlies the *Logic'*s definition of *extension* and its statement of the truth-conditions for categorical propositions.

The Many Faces of Inconsistency

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To think about inconsistencies involves reflecting on several basic notions widely used to talk about human knowledge and actions, such as negation, opposition, denial, assertion, truth, falsity, contradiction and incompatibility, just to name the more perspicuous ones. All of them are regularly used in natural language and for each one of them several definitions or conceptions have been proposed throughout the history of western thought. That being so we tend to think that we have a good enough intuitive understanding of them. Yet a closer examination, as the ones made by Grim (2004) and by myself (Bobenrieth 2003), show many way in which "contradiction" and related word can be understood. Thus, a more precise definition would help to clarify their meaning and assist us to use them in a more appropriate manner. I this presentation I will try to clarify these notions and thus make a terminological proposal. The general background will be the reflexion on paraconsistency. A main purpose will be to show that the confusion between contraries and contradictories -- although they were clearly distinguished in the original square of opposition-- is very common and it paves the way to the reject of all forms of "inconsistencies" without making distinctions, and to the wrong assumption that regarding all the main aspects the effects of contrary opposition are equivalent to the ones of contradictory opposition.

A Morphogenetic Map of the 'Hybrid Hexagons' of the Oppositional Tetrahexahedron

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After defining the notion of "hybrid oppositional hexagon" (or hybrid logical hexagon) we show that the oppositional tetrahexahedon of Sauriol, Pellissier and Smessaert contains - unseen until know - hundreds of such hybrid hexagons. We study their qualitative typology, first by determining all the possible shapes and decorations of hybrid hexagons inside the oppositional tetrahexahedron and then by drawing a map of their mutual direct transformations by smallest deformations, which gives a 3D "morphogenetic map". Finally, we argue that the latter helps considerably the task of deriving, from "oppositional geometry", an "oppositional dynamics", useful for studying, through oppositional structures, dynamical oppositional phenomena.

Using Square of Opposition to Measure the Justification Level of our Opinions

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By “opinion” I understand the assignment of a truth value to a proposition. For instance, the opinion that “*Aristotle was a Logician*” is equivalent with the assignment of the value *truth* to the proposition “Aristotle was a Logician”. There is possible that an opinion be wrong, when someone assigns a truth value to a proposition which has another value. For example, the belief that “*Paris is the capital city of England*” is erroneous because the *real* value of the proposition “Paris is the capital city of England” is the *false*.

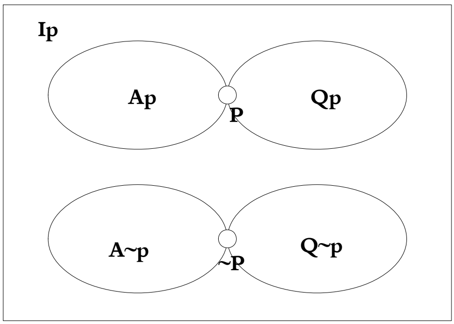
We’ll call *justified* an opinion relatively to a proposition *p* if and only if, the assigned truth value for the proposition *p* is the same with the real value of *p*. The aim of my paper is to define a measure of the justification level of our opinions. If *p* is a proposition, at a given moment and in a given context, we are justified more or less to consider it as true or false. The justification to adopt an opinion can vary from a minimum to a maximum level; the justification of an opinion is *gradual*.

In order to measure the justification level of an opinion it will use the relationships among propositions as they are structured in the *square of opposition*. These relationships split the domain of all propositions into seven classes relatively to a given proposition, *p*:

1. The equivalent propositions with *p*: $\mathbf{P} = \{x/x \leftrightarrow p\}$;

2. The contradictories of *p*: **~P** = {x/x ↔ ~p};
3. The antecedents (supraalterns) of *p*: **Ap** = {x/x → p};
4. The consequents (subalterns) of *p*: **Qp** = {x/p → x};
5. The contraries of *p* (or the antecedents of ~p): **A~p** = {x/x → ~p};
6. The subcontraries of *p* (or the consequents of ~p): **Q~p** = {x/~p → x};
7. The independent propositions relatively to *p*: **Ip**.

These classes are represented in the following diagram (the rectangle represents the set of all factual propositions):



If the following notations are introduced:

|p| = the justification level for certainly believe *p*;
M = the cardinal number of the class **M**,

then, it can be proved that the measure of the justification level to certainly consider *p* as a true proposition is:

$$|p| = \frac{Ap}{Ap+Qp+Ip}.$$

(The justification level to accept the truth of the proposition *p* is proportional with the number of the antecedents of *p*).

It can be also proved several other relations concerning the justification level of our opinions:

$0 \leq |p| \leq 1$
$|p| + |\sim p| \leq 1;$
$|p \text{ v } q| + |p \ \& \ q| = |p| + |q|;$
$(p \to q) \to (|p| \leq |q|)$
$|p \ \& \ q| = [0, \min(|p|, |q|)]$ etc.

Using these formulas we'll be able to find the most justified opinion among a set of given opinions and to rationalize our decisions.

Elucidating Kant's Antinomies with the Hexagon of Opposition

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The antinomies in the *Critique of Pure Reason* consist of four pairs of contradictory propositions concerning fundamental metaphysical questions, such as the infinity or finiteness of the world (First Antinomy), whether there are free causes or not (Third Antinomy), etc. Theses and antitheses of the antinomies are both provable, so reason is unable to avoid contradiction, a situation Kant describes as the death of philosophy. Kant's solution to the antinomies is based on transcendental idealism (TI). Given TI, the alternatives that were contradictory become either both false (in the case of the first two antinomies, the mathematical ones), or both possibly true (in the case of the third and fourth ones, the dynamical).

Commentators have suggested a connection to the square of opposition, in that Kant's solution treats the alternatives of the mathematical antinomies as contraries and those of the dynamical antinomies as subcontraries. We go further, using a general form of the square, suitable for singular terms, such as the the antinomial subject term "the world". A clearer picture of Kant's solution is obtained when the square is extended to a hexagon. The hexagon model shows how the status of the theses-antitheses oppositions can change from contradictory to contrary/subcontrary through the adoption of TI.

On the Hexagonal Organization of Concepts Related to the Ideas of Similarity and Analogy

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Recently, on independent cognitive bases, two hexagons of opposition structuring the interplay between the notions of difference, similarity or analogy and related ideas have been proposed. In the meantime, the idea of analogical proportion ("*a* is to *b* as *c* is to *d*") has been logically formalized as "*a* differs from *b* as *c* differs from *d*, and *b* differs from *a* as *d* differs from *c*". In Boolean terms, it is true iff (*a*,*b*,*c*,*d*) = (1,1,1,1) or (1,0,1,0) or (1,1,0,0) or the 3 companion 4-tuples obtained by exchanging 1 and 0. Another related proportion, called "paralogy", expressing that "what *a* and *b* have in common, *c* and *d* have it also" has the same truth table (invariant when exchanging 1 and 0), except that (1,1,0,0) is replaced by (0,1,1,0). It appears then that Moretti's and Béziau's hexagons may receive a logical read-ing, as shown below, on the basis of 3-partitions of the sets of 6 patterns for which analogy and paralogy are true respectively, which may fit intuition (x stands for 0 or 1, and x' = 1 – x). The two 3-partitions correspond to the vertices of the hexagon traditionnally called **A**, **E**, **Y**:
Moretti's hexagon. **U**: difference: (x, x', x, x') ; (x, x, x', x'). **A**: dissimilarity: (x, x', x, x'). **E**: contrariety: (x, x, x', x'). **I**: non contrariety (x, x, x, x) ; (x, x', x, x'). **O**: similarity: (x, x, x, x) ; (x, x, x', x'). **Y**: sameness: (x, x, x, x).
Béziau's hexagon. **U**: non analogy: (x, x, x, x) ; (x, x', x', x). **A**: opposition: (x, x', x', x). **E**: identity: (x, x, x, x). **I**:

difference (x, x’, x’, x) ; (x, x’, x, x’). **O**: similarity: (x, x, x, x) ; (x, x’, x, x’). **Y**: analogy: (x, x’, x, x’).
As can be checked, these two hexagons are induced by decomposing analogy and paralogy truth tables. The hexagons show the patterns for which the corresponding vertices are true. Note that the notions of ‘similarity’ and ‘difference’ partially differ in the two hexagons.

Is There a Logic? Phenomenology of a Press Review

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The main idea behind this effort is to investigate "the logic" of something and apply it to a real, practical issue. A big help in this comes from Algirdas Greimas' revisit of the Aristotelian square of oppositions that allows a visual representation of the logical articulation of a given semantic category.

Greimas suggests that it is possible to outline reality's descriptive categories in order to find out a "constellation of terms" which is capable, even though contraddictory and opposite, to better characterize reality itself.

The final aim is to demonstrate logic's humble but socially useful purpose. In particular, we are going to deal with a free press review that was distributed in a Milan basilica during the first semester of 2008, called "La nostra rassegna stampa", which had some peculiar features that we investigated.

After learning a little about the publisher, and reviewing several issues set in the general context of the press of the time, we will apply Greimas' – as well as Floch's – ideas and potentiality and evaluate the results. Then, we will compare them with the local features that identify the place in which the revivew was distributed, in order to test its compatibility. In the name of logic.

A Dynamification of Opposition

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A dynamification of logical opposition Op is proposed by means of a functional calculus of opposites O, such that Op(α, ψ) = Op(α,op(α)). To begin with, op amounts to an extensional or intensional negation depending upon whether it proceeds as a function or a multifunction. Then an abstract operator of opposition is devised with the help of O, where subalternation is the counterpart of logical consequence and can be embraced in a broader set of properties.

Finally, the dynamic turn of oppositions is exemplified by an arithmetization of opposite terms that translates these as integers, opposite-forming operators as operators, and oppositions as types of operations between opposites. Another way to describe this dynamification is by introducing vector space theory and Chasles's relation within the geometry of opposition.

Such a process of dynamification is in position to approach two concrete examples of functional opposites, whether in balance social theory or in the familial relationships: my enemy's enemy needn't be my friend, whereas my father's sister must but my aunt. The intensional character of the first example accounts for the non-bipolarity of conflict situations (not everyone is related friendlily or unfriendlily to each other) and makes enemies closer to contraries than contradictories; whereas the extensional character of fatherhood does account for the connection with the contradictory-forming operator.

Algebraic Representations of Syllogistic and the Square of Opposition for the Logic of Information

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Mathematical model of information developed from its conceptualization in terms of the one-many categorical opposition has been proposed in earlier papers of the author. Its formulation is using the concept of closure spaces and lattices of closed subsets. Within this framework, it is possible to apply to the concept of information an axiomatic form of syllogistic understood as algebraic structure, under some, quite strong restrictions of the type of information.

It is a well known fact that every closure space whose lattice of closed subsets admits an involutive authomorphism can be represented as the closure defined by Galois connection generated by a binary relation defined on the set where closure is defined. The involution allows a definition of syllogistic structure, in similar way as orthocomplementation on a lattice allows a definition of orthogonality.

For the purpose of the study of logic for more general concept of information, it is necessary to consider structures of syllogistic type which do not require restriction of the type of closure operations to those which admit involutive authomorphism.

In the present paper, another class of representations of the algebraic structure associated with syllogistic in terms of binary relations is presented. Thus, here too instead of one standard representation of syllogistic in the Boolean algebra of the subsets of a set, we have a variety of representations in the algebra of binary relations. Every representation of an algebraic structure of syllogistic in the binary relation algebra is associated with corresponding representation of the square of opposition. Moreover, for every algebraic structure of syllogistic type on an infinite set, there exists such representation in the algebra of binary relations.

Although the representations of syllogistic algebraic structures are of interest for their own sake, they are of special interest for the search of logical structures for information, in particular for information considered in the contexts not related to natural or artificial languages.

4-Tuples of Transformation Semigroups Satisfying Condition of Square of Opposition

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By a transformation semigroup (X,S,π) (or briefly (X,S)) we mean a compact Hausdorff topological space X, a topological semigroup S with identity e and continuous map $\pi : X \times S \to X$ ($\pi(x,s) = xs$ $(x \in X, s \in S)$) such that for all $x \in X$ and $s,t \in S$ we have $xe = x$ and $x(st) = (xs)t$. We call transformation semigroup (X,S) minimal if $\overline{xS} = X$ for all $x \in X$. We call transformation semigroup (X,S) point transitive if there exists $x \in X$ with $\overline{xS} = X$. We say 4-tuple $\begin{pmatrix} (X_1,S_1) & (X_2,S_2) \\ (X_3,S_3) & (X_4,S_4) \end{pmatrix}$ satisfies condition of square of opposition if:

1) $\forall x \in X_1 \quad \overline{xS_1} = X_1$, 2) $\forall x \in X_2 \quad \overline{xS_2} \neq X_2$,
3) $\exists x \in X_3 \quad \overline{xS_3} = X_3$, 4) $\exists x \in X_4 \quad \overline{xS_4} \neq X_4$,

in other words:

1) (X_1,S_1) is minimal, 2) (X_2,S_2) is not point transitive,
3) (X_3,S_3) is point transitive, 4) (X_4,S_4) is not minimal.

Let $\mathscr{C}$ denotes the class of all 4-tuples $\begin{pmatrix} (X_1,S_1) & (X_2,S_2) \\ (X_3,S_3) & (X_4,S_4) \end{pmatrix}$ of transformation semigroups and

$\mathscr{S}$ denotes the class of all 4-tuples $\begin{pmatrix} (X_1, S_1) & (X_2, S_2) \\ (X_3, S_3) & (X_4, S_4) \end{pmatrix}$ of transformation semigroups which satisfies condition of square of opposition. It is evident that $\varphi : \mathscr{S} \times \mathscr{S} \to \mathscr{S}$ with

$$\varphi\left(\begin{pmatrix} (X_1, S_1) & (X_2, S_2) \\ (X_3, S_3) & (X_4, S_4) \end{pmatrix}, \begin{pmatrix} (Y_1, T_1) & (Y_2, T_2) \\ (Y_3, T_3) & (Y_4, T_4) \end{pmatrix}\right) =$$

$$\begin{pmatrix} (X_1 \times Y_1, S_1 \times T_1) & (X_2 \times Y_2, S_2 \times T_2) \\ (X_3 \times Y_3, S_3 \times T_3) & (X_4 \times Y_4, S_4 \times T_4) \end{pmatrix}$$

is well-defined. Here we want to study subclasses of $\mathscr{S}$ which are closed under other operations like

$$\left(\begin{pmatrix} (X_1, S_1) & (X_2, S_2) \\ (X_3, S_3) & (X_4, S_4) \end{pmatrix}, \begin{pmatrix} (Y_1, S_1) & (Y_2, S_2) \\ (Y_3, S_3) & (Y_4, S_4) \end{pmatrix}\right) \mapsto$$

$$\begin{pmatrix} (X_1 \times Y_1, S_1) & (X_2 \times Y_2, S_2) \\ (X_3 \times Y_3, S_3) & (X_4 \times Y_4, S_4) \end{pmatrix},$$

and other generalizations.

Interdiction and Silence: a Traditional Reading in the Square of Opposition

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This work presents an epistemological and logical investigation about the conditions of possibility for the existence of language. We try to answer the following question: "Why does language exist, instead of nothing?" In order to do this, we deal with both interdiction and silence as constitutive and founders of language, taking into consideration Milner´s assertion: "everything is not said". In other words: if it were possible to say everything, if an utterance existed which said everything that is to be said, this utterance would be the death of language. It follows that it is necessary that language does not say everything, for anything to be said. Interdiction appears here as an element that cuts off the possibility of saying everything so that something can be said. These statements indicate that the incompleteness of language can be linked to a possible answer to the question proposed above. These issues will be treated through a traditional approach to the square of opposition. It should be noted that interdiction, here, is not the same as prohibited. The notion of interdiction will be explained in the moment of the presentation of this work.

Avaktavyam, Contradiction and Opposition

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India, which has a different culture and language, is one of the oldest civilizations of the world history today. And there is no doubt that predominant way of thinking in India is closely related with occurring philosophical thinking and logic in India.

Today, when we say Indian Philosophy and Indian Logic the first question we think about is, if there is a Western way of philosophical thinking in India and study of logic connected to it. Some say there is not. And some say there is, in some ways. Eventually, this question obviously shows us that the study of different cultures and their ideologies is very important for understanding the interaction between civilizations and to have chance to find the differences, similarities and paralellism between their ways of thinking. In this study, we will discuss about Avaktavyam, which is the main subject of the Jain logical system called saptaghangi, in the framework of its relation between contradiction and opposition. Jainism is one of the most important doctrines of Indian philosophy. In this study, which is done in order to explain what Avaktavyam is, the previous studies on Avaktavyam will be taken in hand and then our opinions on the subject will be discussed.
The importance of this study for us is the belief that this study on Jain and its keystone Avaktavyam will be an important source for us in the future in the field of the studies on comparing philosophical thoughts and logic.

Negative Universals: Understanding the Nall Problem

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An old problem in linguistic theory concerns the (universal) absence of universals quantifiers that are morphologically marked for negation (*nall), whereas such negative prefixation is widely attested for existentials (nobody). A similar observation can be made for connectives: whereas many languages have a word for nor, no language in the world has a word for nand (Horn 1989, Jaspers 2007). In this talk, I argue that all current approaches, which aim at providing a synchronic account, face serious problems and I propose an alternative, diachronic, explanation: even though such negative universals never appear are never lexicalized, there is not formal constraint on them; it is just that in the course of language change, such elements can never be formed as a result of lexical merger of a negation and a universal. In short, I demonstrate that (i) a quantifier nall or a connective nand could only have arrived from a phrase [NEG all] or [NEG and] in a previous stage of the language; and (ii) in such a state a negative marker NEG was only a scope marker, indicating that all/and must be under the scope of negation. Then, finally, I show that such phrases are hardly attested, since adding such a statement would only weaken the semantics of the sentence without such a marker.

L'application du quaterne INRC sur la logique interpropositionnelle chez Piaget

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Piaget a fixé seize connecteurs (opérateurs) logiques qui forment seize opérations interpropositionnelles bien définies (l'affirmation complète, la négation complète, la disjonction exclusive, la disjonction non exclusive ou trilemme, la conjonction,…) et il a reconnu que la théorie de la déduction ne peut être correcte sans l'existence de ces opérations qui font parti d'un système logique propositionnel, dont la base se forme de différentes transformations interpropositionnelles qu'on peut effectuer entre les opérations, en appliquant la dérivation ainsi la réduction, celles ci ne dépendent pas d'une opération ou d'un connecteur propositionnel précis comme le montre Scheffer, ou de deux connecteurs selon Frege, Russell, mais plutôt d'un ensemble de procédures.

Piaget a démontré que ces transformations se font en trois manières distinctes et générales : l'inverse, la réciproque et la corrélative, puis il a proposé quatre quaternes inspirés du carré d'Aristote, reliant les différentes opérations et exprimant les différentes transformations.

An Application of Dual Quantifiers

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The application of the notion of the dual Q^d of the quantifier Q to the semantics of expressions with *the same*, as in (1) is proposed:

(1) Leo and Lea read the same books.

The basic observation is that if (1) is true then the set of books that Leo and Lea did not read is also the same: (2a) and (2b) have the same truth value:

(2a) The books that Leo and Lea read are the same.

(2b) The books that Leo and Lea did not read are the same.

Furthermore, the union of the set of books read by Leo and Lea with the set of books not read by Leo and Lea equals to the set of all books. This leads directly to the description given in (3) which is equivalent to (4) and (5)

(3) $SAME(X, R) = \{Z : Z \in PL \wedge (Z_{nom}(R) \cap X) \cup (Z_{nom}(R') \cap X) = X\}$, where Z
runs over the set of (plural) type $\langle 1 \rangle$ quantifiers.

(4) $SAME(X, R) = \{Z : X \subseteq Z_{nom}(R) \cap Z_{nom}(R')\}$

(5) $SAME(X, R) = \{Z : Z^d_{nom}(R) \cap X \subseteq Z_{nom}(R) \cap X\}$

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