1. An overview

In «Advice on modal logic», Dana Scott wrote that he considered concentration on a system with just one modal operator as one of the biggest mistakes of all in modal logic. Since 1970 multimodal systems have burst out. Those systems leave something to be desired. They fail to capture the way in which modal operators interact. It is precisely the question that Alexandre Costa-Leite has addressed in his thesis. He has systematically explored the interactions between two families of modalities: epistemic modalities and metaphysical modalities. For that purpose he has used a new methodology which rests upon two operations recently developed in model theory: the fusion of models and the product of models.

Combining logic plays a major role in philosophical issues whenever the very formulation of those issues requires statements containing non interdefinable operators. The verification principle is a case in point. It says that true propositions are knowable. The term «knowable» is a complex modality made out of two non interdefinable components: the metaphysical modality expressed by the operator «it is possible that» and the epistemic modality expressed by the operator «it is known that».

Hybrid modalities occur in controverted philosophical theses such as the thesis of the logical skeptics who claim that the only knowable propositions are non contingent propositions and that contingent propositions are merely believable. ACL has set out to forge formal tools to test philosophical statements of that kind. To the question «how can we vindicate logical skepticism?», he replies that three necessary conditions (not sufficient) have to be fulfilled: (1) first an axiomatic system has to be worked which expresses logical skepticism fully (namely by capturing the interaction between the modalities occurring in its formulation), (2) a model-theoretic semantics for interpreting the system has to be spelled out. (3) The axiomatic system needs to be proved sound and complete for the intended interpretation. ACL honestly recognizes that he has only carried out the first two steps described above. Yet the mileage covered is most remarkable. For several systems under considerations, the method for abiding with condition (3) is given.

ACL has laid down quite a few axiomatic systems and model theoretic interpretations which disclose the relationships between the concepts of truth, belief, knowledge, contingency and non contingency. Meanwhile he has deductively established
several logical results highly relevant to important philosophical issues. To that extent he has made a substantial contribution to formal philosophy.

In «On Denoting», Russell says that a logical theory may be tested by its capacity for dealing with puzzles and recommend «to stock the mind with as many puzzles as possible, since they serve much the same purpose as is served by experiments in physical science». Judged from that standpoint, ACL’s dissertation has achieved important results. New and improved solutions have been found by the author to several paradoxes which have been worrying philosophers and logicians for a long time: (a) the knower paradox, (b) the paradox of contingency and (c) the Lenzen paradox.

ACL’s combination of logics culminates at the end of the third chapter where the interaction between three modal operators (knowledge, belief and contingency) is explored both axiomatically and model-theoretically.

In the fourth chapter the author shows how paraconsistency can be accommodated in his framework by defining a paraconsistent negation for a fusion of models. In the last chapter, the relationships between modalities are examined in the wider set-up of universal logic and category theory.

2. Comments on some results

The knower paradox is one of the most challenging puzzles to which modal logicians are confronted to day. It is worth recalling the recipe which can be used to produce it. Our starting point is the basic axiom of epistemic logic («if it is known that φ, then φ ») and this elementary theorem: «If it is known that φ and ψ then it is known that φ and it is known that ψ». Applying the necessitation law of basic modal logic to these theses, you get the bimodal formulas: (1) [] (Kφ ⊃ φ) and (2) [] (K(φ ∧ ψ) ⊃ (Kφ ∧ Kψ)). Now take up a substitution instance of the verification principle (3) φ ⊃ ◊Kφ. Substituting p ∧ ¬Kp for φ, you obtain (4) (p ∧ ¬Kp) ⊃ ◊K(p ∧ ¬Kp). By trivial transformations on (4) you get the collapse principle (5) p ⊃ K p («all true propositions are known ») which identifies truth with knowledge when conjoined to (1).

A considerable amount of authors have striven to block this unwanted consequence. Some brought intuitionistic to bear on the issue. Other built the grammatical distinction between indicative and subjunctive moods into the logical formalization of the verification principle. All these solutions are very complicated. Moreover they are not always well motivated.
In chapter I of his thesis, ACL offered an altogether new approach to the knower paradox. He brought into focus a semantic difference between the logic of possibility and the logic of knowledge which had passed unnoticed. Even if the logical laws governing these two modalities overlap, the accessibility relations which are used to state the semantics of the modal operators ◊ and K differ. This insight led ACL to apply the technique called fusion of models to the bimodal language involved in the formal rendering of the verification principle. With this new conceptual apparatus at his disposal, he managed to produce a countermodel which falsifies the troublesome substitution instance of the verification principle, namely (4), without appealing to a deus ex machina. Rather than solving the paradox, he has dissolved it.

Chapter II provides philosophers with a formal definition of (a) logical skepticism and (b) weak logical skepticism. The first reads as follows: «if a proposition is contingent, it is not known». The second says: «if a proposition is contingent, it is not knowable». Next ACL builds a formal semantics which, besides the standard clause for the interpretation of «ϕ is known», contains new clauses for the interpretation of «ϕ is contingent» and «ϕ is non contingent».

Here again the fecundity of the fusion of frames and models is striking. While he was experimenting with the fusion of the model for knowledge with the model for contingency (respectively non contingency), the author hit upon interactions between knowledge and contingency (respectively non contingency) automatically generated by mere fusion of frames, an intriguing phenomena observed elsewhere by P.Y. Beziau who dubbed it «copulation paradox». ACL should be praised for extracting new axioms of contingency out of these.

The following result is worth stressing at this stage: the conjunction of the apparently innocuous principle of verification with the proposition which states weak skepticism entails that if a proposition is true it is not contingent, a conclusion which is much more startling than any of its two premises. ACL dubbed it «the paradox of contingency». He also derived a baffling conclusion from the statement that contingent propositions are knowable conjoined to the converse of verification principle. This second result however is less interesting. The converse of the verification principle («if ϕ is knowable, then ϕ is true ») is much more dubious than the principle of verification itself. Who would assent to the following statement: «If there is a possible world in which A knows that unicorns exist, then unicorns exist.» ?
In Chapter III, the three modalities knowledge, belief and contingency are brought together in a ternary fusion which operates at three levels: linguistic, axiomatic and model-theoretic. Meanwhile ACL revisits the Lenzen paradox. Applying his new conceptual apparatus, he discloses unnoticed differences between Lenzen’s paradox and Moore’s famous paradoxical sentence: « It is the case that \( p \), but I do not believe that \( p \) ».

The assumption we have to make to get Lenzen’s paradox is very close to Moore’s sentence. It reads as follows: « \( \neg p \) is true, but the agent believes that \( p \) is true ». There is however a major difference. A plain contradiction can be derived from Moore’s assumption within the monomodal logic of belief. On the contrary, as ACL rightly observes, Lenzen’s paradox requires at least a fusion generating a bimodal logic of belief and knowledge to get off the ground.

One should not believe however that the kind of logical skepticism which ACL explores is a speculative position which cannot be sustained seriously. Proponents of logical skepticism downgrade contingent propositions in so far as they contend that those propositions cannot be known. Yet they acknowledge that they can at least be believed. Contingent propositions are sorts of « second class citizens ».

Paraconsistent logics have been invented to preclude theories or knowledge bases which contain hidden contradictions from blowing up by a mere application of the principle « e falso sequitur quodlibet ». Paraconsistent logics have been described by some philosophers of logic as « deviant logics ». Hence it is unexpected and remarkable that ACL succeeded in reconciling a deviant logic (paraconsistent logic \( C_1 \)) with a classical multimodal logic which initially was a conservative extension of propositional logic.

With the appeal to category theory and universal logic, the discussion is pursued at a very high level of abstraction and generality. Yet the author never loses sight of his general purpose. Throughout the dissertation he strives to articulate the metaphysical modalities with the epistemic modalities. Nothing could have condensed his logical journey better than the diagram occurring on p. 169 which discloses all the relations which exist between knowledge, non contingency, necessity, possibility, contingency and belief.

3. Critical remarks

3.1. On p. 37, the way ACL obtains \((\varphi \land \neg K) \supset (\varphi \land \neg \Diamond K \varphi)\) by appealing to Fitch’s theorem 5 is not clear. A much simpler way exists if we avail ourselves of the principles I mentioned above (1), (2) and (3) which I borrow from a recent paper of Helge Rückert.
3.2. On p. 63, ACL writes: «The skeptical thesis according to which there is just knowledge of the a priori and formal statements appeared especially as a consequence of epistemic logic and the epistemic rule of necessitation, which states that if a proposition is proved to be true, then the agent knows such a proposition». I disagree.

The rule of necessitation leads to the conclusion that agent know all the theorems. It does not lead to the conclusion that the agent knows theorems only. ACL’s argument is a non sequitur. This oversight of the author can be corrected easily.

3.3. On p. 84, ACL writes down two proofs of the formula which expresses the proposition: «If p is known, then p is non contingent». The first one of these proofs rests upon the assumption that knowledge implies necessity. That assumption is unwelcome. On p. 72, «ϕ is non contingent» is translated into «either ϕ is necessary or ¬ϕ is necessary». Hence ϕ is necessary entails that ϕ is non contingent but not conversely. In other words, the author takes for granted a proposition that asserts more than what he wants to prove. This is a petitio principii.

Moreover line 4 in the first proof is useless.

On the same p. the author provides a second proof (of the same formula) which is perfect. It suffices to drop the first one.

3.4. On p. 110, ACL writes down a formal proof of Lenzen’s paradox (« all beliefs are true hence the distinction between knowledge and belief collapses »). The proof leading to that baffling result stands or falls depending whether we endorse two questionable principles. The first one is the certainty principle which states that « If an agent believes ϕ he believes that he knows ϕ ». This principle should be discarded. No sensible believer mistakes his beliefs for items of knowledge.

The second dubious principle implicitly used in the proof (between line 10 and 11 of the proof) states that « if an agent believes ϕ and believes ¬ϕ, then he believes ϕ∧¬ϕ ». This principle is no less questionable than the former one. Let us adopt a behaviorist account of belief. We can figure out an agent who behaves as if ϕ and who behaves as if ¬ϕ but we cannot figure out an agent who behaves as if ϕ∧¬ϕ.

4. Objections

On p. 66, ACL writes « [...] considering the scope of the present text, the principle according to which language is a mirror of reality plays an important role ». On p. 85, he endorses the Tractarian thesis even more confidently: « Indeed, being the mirror of the world, a result obtained in a linguistic construction can be exported to reality ». In my
opinion this dependence upon a particular philosophy is a weakness. Logic and formal philosophy should carry no commitment to a given philosophy.

ACL tries to evade the objection by embracing relativism (p.85 again): «a criticism based on different criteria would be the same as [...] playing a different game». This is too easy. Instead of restricting oneself to the study of language (propositional logic in the Tractatus, predicate calculus in Quine’s Word and Object, one might look at theories of modern physics for information about the structure of reality.

This alternative approach may lead us to recognize that mathematics cannot be separated from physics and that mathematical structures impose constraints of their own to the form of the laws of nature.

This divergence with the author is of minor importance.

5. Questions

5.1. What is the intuition behind axiom 2 of the logic with contingency (p.71, bottom)?

5.2. How do the two axioms listed in the middle of p.82 automatically emerge out of the interaction of modalities?

6. Overall assessment

There are several papers dealing with the relation between epistemic possibilities and agents’ abilities. They belong to the Belief-Desire-Intention paradigm (BDI) or to the Knowledge-Ability-Result-Opportunity paradigm (KARO). The papers belonging to those traditions deal with a sense of «possible» which is often expressed by the auxiliary verb «can». This sense is altogether different from the sense considered by ACL.

To my knowledge, ACL is the first, with the exception of H.von Wright, to have tried to formalize the relationship between the epistemic notions of knowledge and belief with the metaphysical notions of contingency and necessity. Hence ACL fills an important gap in the logico-philosophical literature.

The bulk of new axiomatic systems and new semantics laid down in the dissertation is impressive. Far from closing up a subject by an exhaustive examination, the author has opened a new road to formal philosophers. Up to now the technical methods which are needed to combine different modal systems (fusion, product and fibring) had remained a research field for advanced logic. ACL has to be credited with the merit of applying these methods to the solution of open problems and paradoxes which develop in the foreground of philosophical discussion. The work presented is an innovative and deep contribution to both
logic and philosophy. It contains a rich harvest of results and paves the way for further researches. It meets the requirements of a Ph.D. eminently. I fully agree that the thesis deserves to be defended on July 3.

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