# Logical oppositions in Arabic logic

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#### Introduction

- Arabic logicians are often seen as faithful to Aristotle's logical work since much of their work consisted mainly in explanations and commentaries on Aristotle's *Organon* and what followed it. But does this mean that there is no significant difference between them and Aristotle?
- To answer this question, I will study Avicenna's and Averroes' views on logical oppositions and compare between their theories and Aristotle's. I will focus on the way logical oppositions are characterized in their respective systems, on their respective views about the notion of opposition and the differences between these views and Aristotle's one.

#### Avicenna's analysis (980-1037)

- Logical oppositions are analyzed by Avicenna in Al Maqūlāt (Categories) and Al <sup>c</sup>Ibāra (Peri Hermeneias).
- In Al <sup>c</sup>Ibāra, he considers the four traditional oppositions: contradiction, contrariety, sub-contrariety and subalternation
- Opposition is defined as follows: "Opposites do not combine in the same subject by any aspect in any time" (Al Maqūlāt, p241). It may concern terms or propositions.
- Following Aristotle's *Categories* (10), he distinguishes between correlation ("double" and "half"), possession and privation, contrariety ("sick" and "healthy") and the opposition of truth values. We will focus here on this last opposition.

#### Avicenna (980-1037)

- The opposition in truth values is introduced by negation since sentences that contain opposite predicates such as "sick" and "healthy" are not contradictories when the subject does not exist, because in that case, they are both false. In the same way, the sentence "Stones are sick" does not contradict "Stones are healthy" but it does contradict "Stones are not sick" because the first two sentences are both false while the last one is true.
- But the notion of opposition is more general than contradiction, contrariety or correlation. It can be seen as a "genus that we can subdivide into several species" (Al Maqūlāt, p245).

- In Al <sup>c</sup>Ibāra (Peri Hermeneias), Avicenna classifies propositions into three kinds: 1) singular, 2) indefinite (= non quantified) and 3) quantified.
- The quantified propositions are universal or particular. Avicenna uses **specific words** to express quantification. The **quantifier** is expressed by the word "*sour*" and the quantified propositions by "*mousaŵara*". This explicit distinction between the quantifier and the terms, which are the subject and the predicate, **prefigures** the medieval distinction between *syncategorematic* words and *categorematic* words.

• **Singular** propositions are **contradictories**, that is, they never share the same truth value.

But in *Al <sup>c</sup>Ibāra*, Avicenna says that the singular propositions which are in the future are not necessarily true or false (p 70); he seems then to agree with Aristotle in his treatment of the problem of the "future contingents", although he gives more details on the possible propositions.

Quantified and non quantified propositions are opposed in many ways, but their oppositions are all related with the notion of truth value.

- When we add negations, the universal negative is expressed by "No As are Bs" and the particular negative by "Not all As are Bs".
- Avicenna considers the two pairs E-I and A-O as contradictories. But what is added is the subdivision of these quantified propositions into three kinds which are: Necessary, Impossible and Possible.
- Necessary, Possible and Impossible must be understood in terms of the relation between the subject and the predicate. The necessity, possibility or impossibility are internal and are not expressed by a specific word.

- As examples, Avicenna gives the following:
  - Necessary proposition: "Every man is an animal": It is necessary because of the fact that being an animal is an essential attribute of men.
  - Impossible proposition: "No man is a stone". It expresses the fact that "stone" cannot be one of the features of the subject "man ".
  - Possible proposition: "Men are writers": the predicate "writer" might apply to the subject "man" but when we add "all" or "no", we obtain two false propositions which are: "All men are writers" and "No man is a writer".

- These modalities are called "matter" modalities because they express the inherence of the predicate into the subject and are related to the essences of the objects concerned. When this inherence is permanent, the proposition is necessary, when it is not, the proposition is possible, when the predicate can never apply to the subject, the proposition is impossible.
- They must be distinguished from the explicit modalities which are expressed by specific words such as "necessary" (= wa'jeb), "possible" (= momkin) and "impossible" (= momtana<sup>c</sup>a). A sentence which contains an *explicit modality* could be false as with the example "All men are necessarily writers", while a sentence which contains a matter modality is never false when it is necessary and affirmative (or impossible and negative).

- "Matter" modalities are considered in the general theory of (categorical) syllogisms while explicit modalities are studied in the theory of modal syllogisms. These "matter" modalities are comparable to the medieval notions "materia necessaria", "materia contingenti", and "materia impossibili" which are mentioned by Pacius (See Aristotle, De l'interprétation, note 5, p 110) and commonly discussed in medieval times (See Knuutila, 2008).
- The examples given make it possible to distribute the truth values of these propositions in the following way:
  - A necessary = true, A impossible = false, A possible = false.
  - E necessary = false, E impossible = true, E possible = false.
  - I necessary = true, I impossible = false, I possible = true
  - O necessary = false, O impossible = true, O possible = true.

A Necessary : TRUE	E Necessary: FALSE
A Impossible: FALSE	E Impossible: TRUE
A Possible : FALSE	E Possible: FALSE
I Necessary : TRUE	O Necessary: FALSE
I Impossible: FALSE	O Impossible: TRUE
l Possible : TRUE	O Possible: TRUE

- As the table shows, contradictory propositions are the following six pairs: 1) Necessary A and Necessary O, 2) Impossible A and Impossible O, 3) Possible A and Possible O, 4) Necessary E and Necessary I, 5) Impossible E and Impossible I, and 6) Possible E and Possible I.
- All of these are **contradictions** since they never share the same truth-value. Avicenna says exactly which one is true and which one is false in all cases. They are then **totally opposed whatever matter they may have** as he notes in chapter 10 (pp 66-75). This makes contradiction the strongest kind of opposition.

- The second kind of opposition is contrariety. Contrary propositions don't share the same value in the necessary and the impossible since Necessary A: "Every man is an animal" is true while Necessary E: "No man is an animal" is false. Impossible A: "Every man is a stone" is false while Impossible E: "No man is a stone" is true. But they are both false when they are Possible, since Possible A: "Every man is a writer" and Possible E: "No man is a writer" are both false.
- Contrariety is defined in the traditional way as the propositions that are never true together but might be false together, but the cases of truth and falsity are determined exactly. It is less strong than contradiction because A and E are opposed only in two matters.

- Subcontrary propositions are I and O. As the table shows, these do not share the same truth value in the Necessary and the Impossible, but they are both true when they are Possible. Thus, I Necessary: "Some men are animals" is true while O Necessary: "Not all men are animals" is false, and I Impossible: "Some men are stones" is false while O Impossible "Not all men are stones" is true. Possible particulars are exemplified by: "Some men are writers" and "Not all men are writers" which are both true.
- This is even less strong than contrariety since we have two propositions opposed in two matters but they are true in the third one.

- Subaltern propositions, i.e. the two pairs A I and E O are called "moutadākhila". They are opposed in the possible since A possible: "Every man is a writer" is false while I possible: "Some men are writers" is true; E possible: "No man is a writer" is false, while O possible: "Not all men are writers" is true. But they do share the same truth value when they are Necessary and when they are Impossible.
- What is interesting here is the way of expressing this opposition since Avicenna says:

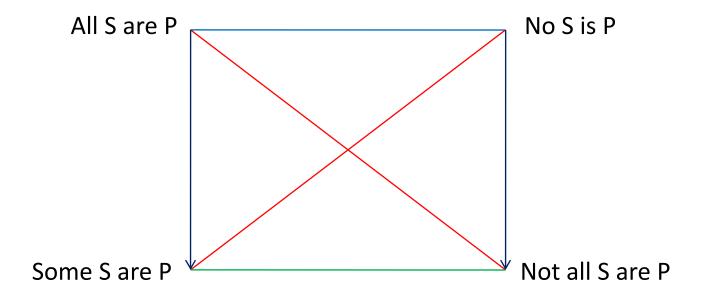
"As to those of the same quality but not the same quantity, *let us call them* Subalterns, we *find* that those which are affirmative are true in the Necessary, and that the negative Subalterns are true in the Impossible, and both do not share the same truth value in the Possible, but the particulars are true in that case, and examine that by yourself " (*Al <sup>1</sup>bāra*, p 48; my emphasis and translation)

- As we can see, his characterization of oppositions follows from the distribution of truth values. This makes his method more semantic than Aristotle's since it is based on that distribution of truth values which follows itself from the meanings of the sentences.
- We can add that the word "Tadākhol" which corresponds to subalternation is not synonymous with the word "Subalternation", since it does not have the same linguistic meaning. While 'Subalternation' derives from the latin words 'alter' which means 'other' and 'sub' which means 'under' and evokes the notion of dependence (upon the other) and thus implication, the Arabic word comes from the root "dakhala" which is a verb meaning 'to enter' and from the closer verb "tadākhala" which means "to enter into each other". It evokes then the idea of inclusion (of the part into the whole).

- Oppositions involve more or less differences in truth values, but the differences are either total or partial i.e. concern only some cases. The strongest opposition is contradiction since it involves all pairs in all matters but the other ones are different in degree so that we can say that subalternation is the less strong one while contrariety which involves one pair of propositions and two modes (the propositions being false in the third mode) and subcontrariety, which involves also one pair of propositions and two modes, are intermediates.
- As we can see, the oppositions between quantified propositions lead to a **Square of oppositions** in Avicenna's view since he admits the four oppositions.

- **Indefinites** should be seen as particulars even though they do not contain explicitly any quantification.
- But what happens if we negate an indefinite proposition? According to Avicenna, the negative indefinite is **not** the **contradictory** of its corresponding affirmative. He claims: "the indefinite has no contradictory" and also: the two indefinites are "**subcontraries**" (p 67) because they are particulars.
- But this can be doubted since if we consider the indefinite as a particular, 1) it should behave as such in all circumstances, which is not warranted, 2) the particular has a contradictory which is the universal negative and this does not fit with what Avicenna says about the fact that the indefinite has no contradictory and makes his opinion on this kind of propositions not very convincing.

- We could add that it is confusing too because Avicenna does take examples of indefinite sentences that he considers explicitly as contradictories, namely the following two sentences: "Stones are sick" and "Stones are not sick".
- The shape corresponding to this analysis of the opposition is then a square since the four oppositions are admitted and the indefinite is not characterized with enough precision. This square is the following, where the red symbolizes contradiction, the blue represents contrariety, the green represents sub-contrariety and the black represents subalternation.



#### Averroes' analysis (1126-1198)

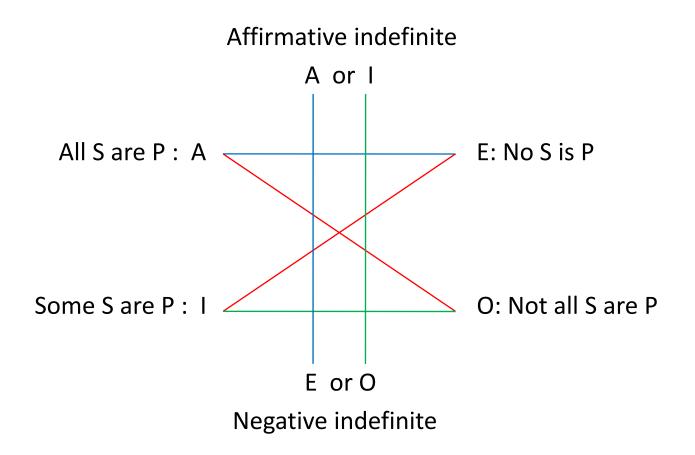
 Regarding Averroes, things are different because his aim was explicitly to explain and comment on Aristotle's Organon. His writings have been grouped and published in 1982 by Gérard Jehamy under the title: Talkhis Manteg Aristou. The book contains the following texts of Averroes: 1) Thalkhīs Kitāb al Maqūlāt [Paraphrase of the Categories], 2) Talkhīs Kitāb al <sup>c</sup>Ibāra [Paraphrase of the Peri Hermeneias], 3) Talkhīs Kitāb al Analitīqa al Awel (or al Qiyās) [Paraphrase of the Prior Analytics], 4) Talkhīs Kitāb al Analitīga athāni (or Al Burhān) [Paraphrase of the Posterior Analytics], 5) Talkhīs Kitāb Al Jadal [Paraphrase of The Topics], 6) Talkhīss Kitāb al *Moughālata* [Paraphrase of the *Sophistical refutations*].

- Averroes follows Aristotle's text faithfully.
- But this does not mean that his opinions are exactly similar to Aristotle's. For it happens to him to diverge from Aristotle's text even if his aim is to explain it. For instance, he writes sometimes "He (i.e. Aristotle) says... and we say...", which shows that when commenting the text, he gives his opinion on it.
- The differences concern also the titles since chapter 7 is entitled: "On the definition of the universal and the particular and the explication of the universal and the particular quantifiers and the determination of the six oppositions", while chapter 7 in the *Peri Hermeneias* is entitled simply: "Universal and Singular. The opposition of propositions: contradiction and contrariety".

- According to Averroes there are six oppositions between: 1) The singular propositions, 2) The indefinite propositions, 3) The quantified propositions.
- The opposition between the two singular propositions is a **contradiction**.
- The opposition between the two indefinites or non quantified propositions is either **contrariety** when both are universal or **subcontrariety** when they are particular. They are universal when they are **necessary** and particular when they are **possible**. Possibility and necessity are as in Avicenna's view "matter" modalities.
- Ex: "Men are white" and "Men are not white" are **subcontraries** because they are true together.

- If we consider that there are two pairs of contradictories, the total number becomes really six oppositions, that is: 1) Singulars, 2) Indefinites, 3) Contradictories<sub>1</sub>, 4) Contradictories<sub>2</sub>, 5) Contraries, and 6) Subcontraries.
- Contradictories are those which never share the same truth value such as the singular propositions and the quantified propositions which never share the same truth value "in all matters". These are 1) A and O and 2) E and I.
- **Contraries** are the two universals and do not share the same truth value "in the Necessary and the Impossible" but are both false in the Possible. Ex: "Every man is white", "No man is white" are both false.
- **Subcontraries** are the two particulars and are never true or false together in the Impossible or the Necessary but are true together in the Possible.

- However, like Aristotle, Averroes does **not** mention the **Subalterns** which he seems to ignore completely. This might be explained by the fact that he is commenting on the *Peri Hermeneias* in which we don't find any mention of the Subalterns.
- It seems then that he combines between Avicenna's views (by adopting the same classification of matters) and Aristotle's ones by citing the kinds of oppositions. But he differs from Aristotle by considering subcontrariety as a *real* opposition while it is only a *verbal* opposition in Aristotle's view. The other difference between him and the two other authors concerns the indefinites which are considered explicitly as *ambiguous* while in Aristotle's and Avicenna's views they ought to be considered as particular rather than universal. The shape corresponding to this classification is thus the following:



 This shape is different from Aristotle's and Avicenna's ones. It shows that the indefinite might be included inside the square since it has the specificity of being ambiguous and could be assimilated neither to the particular nor to the universal. If we add the singular propositions, we have an even more extended shape which will contain one more horizontal line which in this case would be red. This makes Averroes' position about the notion of opposition itself different from both Avicenna's and Aristotle's ones.

## Differences and affinities between these views

- According to Aristotle, only contradiction and contrariety are oppositions and the shape he admits is just a fragment of the square as has been shown for instance by T. Parsons. Opposition according to him is then understood in the following way:
- Two propositions are **opposed** to each other if and only if:

1) They have the same subject and the same predicate **but** one of them is affirmative and the other is negative

2) Either they never share the same truth value or they are never true together

This shows as J-Y Béziau has noted, that Aristotle "privileges the principle of contradiction over the principle of excluded middle" because contrariety respects the first principle but not the second while subcontrariety respects the second principle, which creates an illegitimate "asymmetry" since both principles are equally admissible. We can even show them to be equivalent by using De Morgan's laws and the law of double negation for instance.

- Regarding Avicenna, opposition is characterized by the difference of truth values. Whenever there is such a difference, there is some kind of opposition; the propositions might then be opposed to each other totally or partially and there is a graduation in the oppositions. The notion of opposition seems then to be plural unlike Aristotle's notion and it goes from the strongest kind to the less strong one which is Subalternation.
- We could then define opposition by distinguishing between a complete opposition and partial oppositions in the following way:

• I/ Two propositions are **opposed completely** if and only if:

1) They have the same subject and the same predicate **but** one of them is affirmative and the other is negative

2) They never share the same truth value whatever matter they have.

II/ Two propositions are **partially opposed** if and only if:

1) They have the same subject and the same predicate **but** one of them may deny the other

2) They do not share the same truth value in one or two matters

This distinction that I make between the two kinds of oppositions is justified by the fact that Avicenna considers contradiction as the most important opposition and that he says that opposition is "a genus which could be subdivided into species" (*AI Maqūlāt*, p 245)

- Avicenna agrees with Aristotle in giving a great importance to the principle of contradiction since he devotes a whole chapter (chapter 10 of Al <sup>c</sup>Ibārā) to the notion of contradiction and defines opposition by means of it.
- However, there are some incoherencies in his position especially regarding the indefinites which are considered as particulars and are said to have no contradictory.
- But 1) there is no reason why one kind of propositions could not have a contradictory, 2) if the affirmative indefinite is particular, its negation would be universal, which contradicts the general opinion that the two indefinites must be seen as particulars.
- Moreover, it happens to Avicenna to give examples containing indefinites and to say that they are contradictory as we have seen before.

- Averroes seems to be closer to Aristotle than Avicenna is, for he follows Aristotle's text faithfully and agrees with him in many points for instance in not considering subalternation as an opposition; but the theory he defends about opposition is, in the final analysis, slightly different from Aristotle's theory.
- For he admits subcontrariety as an opposition, which distinguishes him from Aristotle and he uses the same way as Avicenna in classifying the propositions into necessary, possible and impossible. Moreover, he distributes the truth values in the same way as Avicenna. Regarding the indefinites, his opinion is different from both other opinions since he considers it explicitly as ambiguous and not as in Avicenna's and Aristotle's views as a particular.

- We could then define his notion of opposition in the following way:
- Two propositions are **opposed** to each other if and only if:

1) They have the same subject and the same predicate **but** one of them is affirmative and the other is negative

2) Either they never share the same truth value or they are never true together or they are never false together

Opposition is then plural but restricted to three main kinds of oppositions, it is less limited than Aristotle's notion but more limited than Avicenna's one. The reason for that may be that he is not convinced by Aristotle's claim that subcontrariety is a verbal opposition, although he does not comment explicitly on it. But he thinks like Aristotle, that opposition involves a difference in the quality of the propositions concerned.

- But his treatment of the indefinites is not very convincing too. For in his view, the indefinite has either a subcontrary or a contrary proposition depending on its nature; he does not say what is its contradictory and seems to share Avicenna's opinion that it does not have any contradictory. However, when the universal indefinites are necessary or impossible, they are not contraries, but rather contradictories, ex: 'Men are animals' and 'Men are not animals'.
- Only possible indefinites could then be ambiguous (sometimes universal, sometimes particular), but if we say that these indefinites are expressed by  $A \lor I$ , and  $E \lor O$ , there are no new vertices, since if A and I have import,  $(A \lor I) \equiv I$ ; and  $(E \lor O) \equiv O$  whether E and O have both import or not; if we use rather 'W', 'A W I' and 'E W O' become equivalent, when A and I have import, whether E and O have both import or not. This makes his opinion about the indefinites not satisfying despite the fact that he includes them into his theory of oppositions quite explicitly.

- However, despite these differences, they are close to Aristotle regarding the existential import of the universal propositions (especially **A**) and their wording of **O** which is "Not all As are Bs". As Terence Parsons says: "Aristotle's articulation of the O form is *not* the familiar 'Some S is not P' or one of its variants; it is rather 'Not every S is P'. With this wording Aristotle's doctrine automatically escapes the modern criticism" [12] this wording of **O** solves the problem about oppositions raised by the modern logicians because it makes all the relations valid.
- Parson's argument leads to the opinion that "affirmatives have existential import, and negatives do not" (idem).
- But this argument is conclusive just in case E has no import and O is correctly formalized, for if we formalize O as usual (i.e.~(x)(Sx ⊃ Px)) and check the validity of the relations by constructing truth tables, we discover that the traditional A and O are not contradictories.

- This appears clearly when we formalize A by "(∃x)Sx ∧ (x)(Sx ⊃ Px)" (See Kleene who expresses the existential import in that way in: Logique mathématique, p 146) and O by "~ (x) (Sx ⊃ Px)" and construct a truth table by considering that there are only x<sub>1</sub> and x<sub>2</sub> in the universe. The following line of the table:
- { $(Sx_1 \lor Sx_2) \land [(Sx_1 \supset Px_1) \land (Sx_2 \supset Px_2)]$ } $W \sim [(Sx_1 \supset Px_1) \land (Sx_2 \supset Px_2)]$

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shows that there is no contradiction since, as we can see, *there is* a case of falsity under 'W' which means that 'W' which is the exclusive disjunction is invalid.

But if **O** is formalized as follows:  $\sim [(\exists x)Sx \land (x)(Sx \supset Px)]$ , then the contradiction **A**-**O** holds since this formula is indeed the exact negation of **A** when **A** has existential import. This formalization is equivalent to the following one :  $(\sim(\exists x) Sx \lor (\exists x)(Sx \land \sim Px))'$  which corresponds to the way **O** is expressed by Parsons in ([13], p 6). This last reading of **O** is said by Parsons to be the one given by Buridan and Ockham (idem, p5). It makes also the subalternation **E**-**O** valid, when **E** does not have import.

The question is then: does **E** have import in Aristotle's theory? and in Avicenna's and Averroes' theories?

As a matter of fact, neither Aristotle, nor Avicenna or Averroes have said explicitly that **E** should not have existential import, but the three of them have considered *singular* propositions that have a non existent subject as always false when affirmative and true when negative. By generalization, this leads to the opinion that all negative propositions which subject is non existent are true.

But Aristotle's text is not that clear since he says: «J'appelle universelle, l'attribution ou la non-attribution à un sujet pris universellement..." (*Premiers Analytiques*, I, 1 24 a, 17-20), and seems to treat A and E in the same way with regard to their subject. This makes many authors say that E, as well as A, has existential import in his theory. This opinion is reported by Michael Wreen who says: "The chief difference between classical (Aristotelian) logic and modern (Russellian) logic, it's often said, is a difference of existential import. (1) In classical logic, all categorical propositions ("All S is P"; "Some S is P"; and so on) have existential import" ("Existential import", *Revista Hispanoamericana de Filosofia*, vol 16, N° 47, (Aug, 1984), p 59, my emphasis)

The same could be said about Avicenna and Averroes who do not deny explicitly import from **E**. Their theories are then as unclear as Aristotle's, but could escape modern criticisms for the same reasons.

#### Conclusion

- It follows that Avicenna's and Averroes' treatment of the notion of opposition are quite different from each other and different from Aristotle's treatment too.
- This notion appears to be stronger in Aristotle's view, it is considered as plural in Averroes' and Avicenna's views, but these authors differ form each other too in that Avicenna distinguishes a strong opposition which is contradiction and other species which seem to be partial oppositions while Averroes considers that the difference of quality is fundamental and does not admit for this reason subalternation although he does admit sub-contrariety unlike Aristotle.
- Their analysis seems also to prefigure the medieval distinctions and classifications and it is based on a method which we could characterize as semantic since it relies on a distribution of truth values which follows itself from the meanings of the sentences.

#### Conclusion

- This method is different from Aristotle's one which is more deductive.
- Nevertheless, there are affinities with Aristotle and our two Arabic logicians since they are both influenced by him in defining their concepts and constructing their categorical syllogistic, they give obviously import to A, but are as unclear about E as Aristotle, because they do not deny explicitly its import. However, they also could avoid modern criticism if we generalize their treatment of the singular negative propositions which subject is non existent to all kinds of negative propositions.

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### Thank you!