HAND BOOK OF THE SECOND WORLD CONGRESS ON THE SQUARE OF OPPOSITION

Edited by

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1. Second World Congress on the Square of Opposition

1.1 The Square : a Central Object for Thought

The square of opposition is a very famous theme related to Aristotelian logic dealing with the notions of opposition, negation, quantification and proposition. It has been continuously studied by people interested in logic, philosophy and Aristotle during two thousand years. Even Frege, one of the main founders of modern mathematical logic, uses it.

During the 20^{th} century the interest for the square of opposition has been extended to many areas, cognitive science ultimately.

Some people have proposed to replace the square by a triangle, on the other hand the square has been generalized into more complex geometrical objects: hexagons, octagons and even polyhedra and multi-dimensional objects.

1.2 Aim of the Congress

This will be the second world congress organized about the square of opposition after a very succesful first edition organized in Montreux , Switzerland in 2007.

The square will be considered in its various aspects. There will be talks by the best specialists of the square and this will be an interdisciplinary event gathering people from various fields : logic, philosophy, mathematics, psychology, linguistics, anthropology, semiotics. Visual and artistic representations of the square will also be presented. There will be a music show and a movie illustrating the square.

The meeting will end by a final round square table where subalterned people will express their various contrarieties, subcontrarieties and contradictions.

1.3 Scientific Committee

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2 Abstracts of Invited Talks

Comment Profiter du Brouillard entre le Fini et l’Infini?

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Contrairement à l'opinion courante , je ne pense pas que la frontière entre le fini et l'infini soit clairement délimitée . En particulier, le principe de descente infinie est souvent accepté comme logiquement equivalent au principe de récurrence et peut servir de fondement au monde des entiers "naturels". Je voudrais contester le bien-fondé d'une telle affirmation , au vu de l'explosion actuelle des volumes d'information. La taille énorme des échantillons disponibles aujourd'hui remet en cause les paradigmes acceptés en Statistique , et je voudrais utiliser une possibilité analogue en mathématiques "pures" . On peut voir là une version constructive de *l'analyse non standard*, mais cela repose de manière nouvelle le problème de la non-contradiction des systèmes formels axiomatisés, et la nécessité de nouveaux critères de vérité mathématique .

Giordano Bruno est passé par là !

Thinking Outside the Square of Opposition Box

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The traditional Square of Opposition representing logical relations among the four canonical categorical AEIO propositions belonging to Aristotelian syllogistic reasoning is too easily dismissed today as a relic of an outmoded first effort to systematize formal symbolic logical inference. Whether the Square has become merely a historical curiosity depends in large part on whether we think we contemporary logic has nothing more to learn from the Square and the term logic it illustrates, whether or not the Square might still contain any logical surprises or unexploited points of interest. I offer a deliberately naïve series of remarks about the logical relations depicted in the Square, and in particular concerning what once revealed to be conspicuous omissions from the diagram in its display of logical relations. I consider the full range of classical two-term propositions involving three-proposition with total three terms syllogisms in Aristotelian logic with classical propositions containing only positive subject terms and positive or negative predicate terms. It is possible to identify what might be called a *shadow syllogistic logic* in a precisely parallel family of logical relations involving propositions with negated first terms or subject term complements that appear nowhere in the traditional Square. Such propositions, all of which are inversions of the AEIO forms, unexpectedly have no one-way logical inferential (subaltern), contrary, or contradictory logical relations with AEIO propositions whatsoever. The Square of Opposition accordingly remains interesting as much for what it does not show, and the reasons why, as for what it has been chosen to present.

Colours, Squares and Triangles

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In the debate between the defenders of the classical perspective on the logical square of oppositions on the one hand and proponents of a triangular approach on the other, an original compromise was reached in work by Robert Blanché (1969), among others. His proposal amounted to a logical hexagon, in fact a bitriangle consisting of an AYE-triangle of contraries and its subcontraries dual IOU. The latter system was applied to several arguably bitriangular conceptual fields. The hypothesis elaborated here is that there is yet another bitriangle which can be fruitfully mapped onto the hexagon, namely that of the six colours *red*, *green*, *blue*, *yellow*, *magenta* and *cyane* and more specifically of the percepts they embody. It will be shown that the triangle of contraries involves the primary colours *red-green-blue*, with *red* in the A-corner, *green* in the Y-corner and *blue* in the E-corner. The secondary triad *yellow-cyane-magenta* represents the complementaries of the primary colours, a pattern of opposition that will be shown to be the colour equivalent of contradictoriness relations in the logical square (and hexagon) of oppositions. By moving from a Blanché's 2D- star-like model to a color cube, the achromatic colours *white* and *black* will be introduced into the system. Finally, some philosophical implications of the observed isomorphism of logic and colour oppositions will be drawn.

Symmetry and Duality in Fixed-point Calculus

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In 1913, Ernst Zermelo proved that in the game of chess, either White has a strategy to win, or Black has a strategy to win, or both parties have strategies to achieve (at least) a draw. Assuming that a player « survives » whenever she wins or achieves a draw, we obtain the four possibilities of the square of oppositions: A = *White wins*, E = *Black wins*, I = *White survives*, O = *Black survives*. The property holds for many, although not for all games with perfect information. However, Zermelo's theorem can be seen as an example of a more general phenomenon occurring in the fixed-point calculus, which is an extension of the modal logic designed by computer scientists in order to reason about finite or infinite computations. Indeed, the least fixed point captures the ability to win in finite time, while the greatest fixed point captures the ability to survive indefinitely. More specifically, if $F(X)$ is a formula of modal logic positive in X, and $F'(X) = \neg F(\neg X)$ is its dual, we have the following pairs of oppositions: $\mu X.F(X)$ vs. $\nu X.\ F'(X)$, and $\mu X.F'(X)$ vs. $\nu X.F(X)$. (Here, $\mu x.g(x)$ denotes the least solution of an equation $x = g(x)$, and $\nu x.\ g(x)$ its greatest solution.) This duality further refines if we allow formulas with nested fixed-point operators like $\mu X.\nu Y.\mu Z....F(X,Y,Z,...)$. Interestingly, to understand the meaning of such formulas, we need come back to games.

John Buridan's Theory of Consequence and his Octagons of Opposition.

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Medieval logicians extended Aristotle's theory of the syllogism to a general theory of consequence. Much of their work on the modal syllogism was an attempt to correct and clarify Aristotle's theory, the famous problem of the two Barbaras. Within consequence, they distinguished formal from material consequence, and absolute from ut nunc consequence, though the latter was highly contentious. They also developed a theory of properties of terms, in particular, of signification, supposition and ampliation, which played a central role in their theory of consequence. John Buridan's theory of truth, of consequence and of the syllogism was the most sophisticated and extensive among medieval logicians in its theoretical organisation and insight. In particular, Buridan constructed a series of octagons of opposition, dealing both with modal syllogisms and with syllogisms with oblique terms (e.g. genitives), with a surprising and illuminating set of connections between them. However, Buridan's account of the ampliation of modal terms in modal propositions is highly implausible. Arguably, the analogy between temporal and modal propositions is misleading, as shown by William Ockham. But there is still much to be learned from Buridan's careful analysis of the logic of modal and oblique terms.

The Right Square

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The various laws of the syllogism that have come down to us were not drawn just from Aristotle's treatment, but from that amalgamated with sometimes contradictory amendments from thinkers in medieval times. As a result the corpus was easy to confute and surpass, when Frege's predicate logic started to get established in the early decades of the twentieth century. The final death blow to the Aristotelian tradition came when Peter Strawson published his book on logical theory in 1952, since he showed in close detail that each of several ways of interpreting the Aristotelian four forms, in terms of modern predicate logic, could not substantiate all of the traditional rules. Remarkably, Strawson did not consider Aristotle's own way of understanding the four forms, as was pointed out immediately in Manley Thompson's 'On Aristotle's Square of Opposition' Philosophical Review 62 (1953), pp. 251-265. This paper develops in several ways Aristotle's original view of the existential import of the four forms, showing in particular how it can be generalised to cover quantifiers other than the standard universal and particular ones, and how it can be extended to handle relations - the main distinctive feature that was held to make Fregean predicate logic preferable.

3 Abstracts of Contributors

Leibniz, Modal Logic and Possible Worlds Semantics

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Even if Leibniz had no opportunity to conceive an actual modal logic, the fact remains that he worked out a modal metaphysics, of which the inaugural act was, in his *Elementa juris naturalis* (circa 1671), an implicit reference to the Apulean square of opposition, then acknowledged as probably the first sketch of a deontic logic of norms [1]. Afterwards, some scholars wondered whether Leibniz could be « a sort of grandfather of possible worlds semantics for modal logic » [2]. It seems that Leibniz had available resources, particularly for building the S5 modal system. And his awareness of the K distributive axiom $\Box (p \supset q) \supset (\Box p \supset \Box q)$, common to the main modal systems as we know, dissuaded him to trust the easy solution of the necessitarianism grounded on the well-known distinction coming from Boethius between *necessitas consequentiae* and *necessitas consequentis*.

A new modality square may be drawn, according to which each modality in the corner is expressed with quantifications on possible worlds. So possible worlds semantics could supersedes Leibniz's own explanation of contingency, to the satisfaction of those who did not have any confidence in his proof-theory solution style (for a contingent proposition, the reducibility of the predicate to its subject cannot be achieved) even though much-admired by A. Tarski [3]. But from a Leibnizian outlook, Kripke's semantics as well as Lewis' counterpart theory arise some difficulties we intend to precise.

[1] LEIBNIZ G. W.: *Sämtliche Schriften und Briefe*, Reihe VI, Bd. 1, 465-480 ; Von Wright G. H.: « On the Logic of Norms and Action », in Hilpinen R., *New Studies in Deontic Logic*, Dordrecht-Boston, Reidel, 1981 ; Kalinowski G. : « La logique juridique de Leibniz », *Studia Leibnitiana*, IX, 1977.
[2] ADAMS R. M.: *Leibniz. Determinist, Theist, Idealist*, Oxford University Press, 1994, p. 9.
[3] TARSKI A.: « Some observations on the concepts of ω-consistency and ω-completeness », in *Logic, Semantics, Metamathematics: Papers from 1923-1938*, 1983, Hackett Pub. Co., pp. 279-295.

Squaring *any*

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Linguists working with the Square of Oppositions have not or insufficiently dealt with the fact that languages may have two existential quantifiers, a specific and a non-specific one.

English is a case in point. Next to specific *some* English has the non-specific *any*, illustrated in (1) with *–body* compounds.

(1) a. Did you see *somebody*? b. Did you see *anybody*?

Together with negation, *anybody* forms an alternative for the zero quantifier *nobody*.

(2) a. I saw *nobody*. b. I didn't see *anybody*.

Further, at least in English, *any* also have a quasi-universal use, i.e. the so-called 'free choice' use, which can also be negated (in particular when joined by *just*, *old* or both).

(3) *a.* *Anybody* can do that. b. *Everybody* can do that.

(4) GM is *not* just *any* old company.

This paper addresses the challenge with a hypothesis with the following two properties: (i) it starts from the three-layered 'Neo-Aristotelian' square offered by J. van der Auwera (*Journal of Semantics* 13 (1996) 181-195), and (ii) it adds a partial 'three-dimensionalization' (cp. H. Smessaert, *Logica Universalis* 3 (2009) 303-332).

A Square of Prediction :
Stoïcism, Mergaric and Epicurian School and New Academy

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In his *De Fato* Cicero theories by Chrysippus (stoïcian), Diodorus (Megarian), Epicurus and Carneades. The main topic is about affirmations about the future i.e. predictions.Each of these four philosophers, except Epicurus, accepts *Bivalence* (of two contradictories, one is true, the orther false). As for Epicurus, he rejects *bivalence* but admits *Excluded Middle* (the disjonction of two contradictories is true). Chrysippus accepts *Bivalence* because of universal causality (every event is causaly determined so that every truth is true from eternity). Semantics is causally determined from all eternity. Diodorus considers that what is *possible is what is the case or what will be the case*, so that the events are to be what they will be from all eternity. The world is determined by the semantics. Carneades thinks that the eternal truth (or falsity) of assertions does not conflict with indeterminism. His view on semantics is subalternous to that of Diodorus and contradictory to that of Chrysippus. As for Epicurus, the truth of affirmations about the future depends on causality (so that his position is subalternous to that of Chrysippus), but – because he his an indeterminist – he cannot suscribe to *Bivalence* i.e. to a timeless semantics

Alogon and the Logic of Truth in Aristotle's Poetics

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Nous interrogerons ici la relation entre la logique de la *non-contradiction* et le champ poétique tel qu'Aristote l'a analysé et pour lequel il a prescrit des « principes » : outre la difficulté à penser ce que signifie la non-contradiction dans la dimension du désir humain affronté à la réalité, dans celle de l'ignorance et des croyances d'un personnage, dans l'articulation entre action et raisonnements ou pensée tendus entre contingence, possibilité et nécessité, on peut interroger avec Aristote ce qui peut justifier en poétique le recours à de l'*illogique (alogon), de l'absurde (atopon),* des *paralogismes* ou « faux-raisonnements » comme conditions de production de la vérité propre au poétique : ce ne sont pas là seulement des moyens artificiels et trompeurs d'induire la « croyance » ou l'adhésion affective du spectateur, mais bien des modalités *propres* d'une rationalité de la chose poétique? L'idée de nécessités *relatives* à une forme spécifique d'intelligibilité est elle pertinente? Lorsqu'on articule entre eux des actes, des accidents, des rebondissements, toutes les *péripéties* des écarts entre représentations et réalité, une pluralité des régimes de *nécessité* et de possibilité semble s'imposer : l'improbable mais possible néanmoins, le *vraisemblable* mais pourtant irrationnel, ou l'absurde mais *significatif*, peuvent rendre plus perceptible le sens de l'action humaine. Si la nécessité interne d'une histoire, d'une rationalité *articulée* du sens semble autoriser la tension entre la *logique* et son (ses) autre(s), qu'est-ce que la vérité poétique?

Four-Playered Semantics for a Family of Paraconsistent and/or Paracomplete Logics

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In [J. Hintikka & J. Kulas. The Game of Language, D. Reidel Publishing, 1983] was introduced two-playered semantics for classical logic, by viewing it as a game between two players, one of them trying to prove a formula P, and the other one trying to prove its negation, not P. This metaphor provides a nice way for understanding the interplay among the connectives and quantifiers, specially the behavior of negation and implication. However, when dealing with certain non classical logics, we observe that a semantics with four players (instead of only two) is more intuitive and allows for technical advantages which shed some light upon the definition of some non classical connectives. A generalization of Hintikka's Game Theoretical Semantics, with four players, which constitutes a robust technical tool for providing semantics for logics in general, is proposed. In particular, we show formalizations for a family of paraconsistent and/or paracomplete logics, descending from the calculi C_1, P_1 and N_1 of Newton da Costa, using four players, which present some important improvements over the semantics with only two players. In any case, for each of the logics of this family, the four players compose a square of opposition.

The "Numerical Segment": a Useful Paradigm for Classical, Fuzzy and Paraconsistent Logics.

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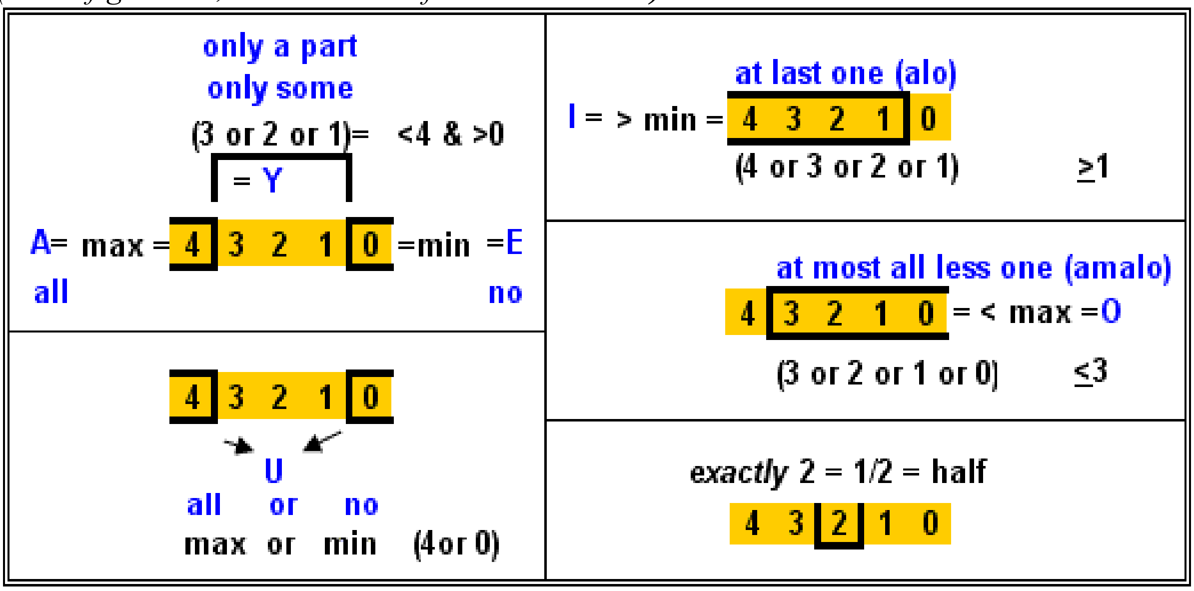
Aristotle (*Metaphysics* X.4, 1055a19–23) draws an interesting geometrical parallel: just as the *extremes of a distance* (διαστημα) are the points farthest removed from each other, in like manner the contraries (εναντιαι) represent the maximal (perfect or complete) difference [of quality].

In this paper we propose, elevating Aristotle's parallel to a real isomorphism, a simplified model of the classic Square or its extensions (e.g. Blanché's hexagon): the *"numerical-distinctive" segment*. This model allows for the representation of the intermediate (partial Y, numerically definite, etc.) categories. These are interpretable as points or subsegments of the segment. The relations of contrariety, subcontrariety and others are redefined in terms of topogeometrical criteria (superimposition, symmetry, inclusion of extremes, etc.). This leads to a rethinking of some basic logical concepts: 1. A reconsidering of the structural correspondences between *quantifiers* and *truth values*, leading to a *unitary ('interbivalent') model* for *classical logics* and for *fuzzy logics*, while further oppositive models are reserved for trivalent or plurivalent logics. 2. A distinction between the *excluded middle* and the *excuded third*. 3. A n-dimensional combining of numerical segments, which transcend the antinomy between *logics of noncontradiction* and *paraconsistent logics*, falling back on probabilistic and modal concepts.

(In the fig. below, "alo" is taken from J.-Y. Béziau).



Logical Oppositions in Arabic Logic

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Arabic logic is considered generally as very influenced by Aristotle's views but when one studies the texts, one can find differences between Aristotle and some of the main arabic logicians such as Avicenna and Averroes. In this paper, I examine the logical oppositions in

Avicenna's and Averroes' texts in order to compare between their respective views on the one hand and between both of them and Aristotle on the other hand. I will rely directly on the arabic texts which are Ibn Sina:"Al-shifâ: Al Mantik, volumes 1& 2" and Ibn Rochd: "Talkhis Mantik Aristou" (Paraphrase de la logique d'Aristote). The square they present is an extension of Aristotle's square since they both distinguish between three kinds of A and E propositions (possible, impossible and necessary) and explicitly say that the A and E although never true together do not share the same truth value when they are necessary or impossible; nevertheless, A and E remain contrary and do not become contradictory since the contradictory propositions are the ones which never share the same truth value in any case. The same thing is true with the I and O propositions which are subcontraries and therefore never false together but may be also possible, impossible or necessary and do not share the same truth value in the last two cases. They also give a special attention to the indefinite which in Averroes' view may be sometimes universal and sometimes particular while Avicenna prefers considering it as a particular even though it appears to be in some cases universal or even singular. The singular propositions are contradictories but their relations with other kinds of propositions are not fully clarified. The logical oppositions are not therefore exactly the same in Averroes' and Avicenna's view since the indefinite is ambiguous in Averroes' picture and may thus be expressed by 'A or I' when it is affirmative and by 'E or O' when it is negative which leads to an hexagon.

Two Kinds of Logic Incompatibility in Law

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1. The deontic square of opposition defines three relationships of incompatibility between norms (antinomy). It is well-known that one of these relationships holds between contraries (*first relationship*: “*p* is obligatory” and “*p* is forbidden”) and the other two hold between contradictiories (*second relationship*: “*p* is obligatory” and “*p* is facultative”; *third relationship*: “*p* is forbidden” and “*p* is permitted”).
2. In the world of *Sein* we know that two propositions are incompatible when they *cannot both* be *true*. But, in the world of *Sollen,* what does incompatibility of norms consist of? Can we find an analogous answer?
Some philosophers have applied the principle of contradiction to the *normative* world and have maintained that two incompatible norms *cannot both* be *valid*. According to others, two incompatible norms *cannot* be *both obeyed*.
3. I criticize both answers and believe that the answer cannot be the same for the three relationships. We need to distinguish between two kinds of opposition.
In the first relationship (contrariety) there is incompatibility at the level of the *agent*: it is impossible for the agent *to fulfil* both norms (*proheretic opposition*).
In the second and third relationship (contradiction), there is incompatibility at the level of the *judge*: it is impossible for the judge *to apply* both norms (*dikastic opposition*).

Boethius' Interpretation of the Square of Opposition in his Syllogistic Treatises

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Boethius in two of his treatises (*De syllogismo categorico* and the *Introductio ad syllogismos categoricos*) presents a detailed treatment of the logical relations of the Square of Opposition by including aspects which are not textually in Aristotle. There are two aspects that are outstanding in his exposition. The first aspect is that he tries to define the Square by means of a traditional division of the categorical proposition, according to which a proposition does or does not have terms in common. Propositions without any common term cannot be studied by formal logic, because they cannot be related, but if a proposition has a common term, then they can be inferred. The Square of Opposition is a branch of this division: those propositions having both terms in common and maintaining the same order, vgr. "Every man is an animal" – "No man is an animal". The division also defines other two branches in correspondence with the doctrine of conversion and the doctrine of syllogisms, which are the other two essential parts of Aristotelian formal logic. Thus, the division gives unity to the general theory and its teaching. The second aspect relates to semantics. Boethius accepts the ancient doctrine of certain matters of propositions (*materiebus*) to justify the validity of the inferences between propositions (and so the relations of the Square) on the basis that the truth value is preserved when the inference is confirmed in every matter (sc. possible, impossible and necessary). Since Boethius is taking over a Peripatetic Greek material to write these treatises, the paper suggests that Boethius in doing so gives the most original and traditional exposition of the Square of Opposition.

The Square of Opposition and Generalized Quantifiers

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A relevant aspect of the meaning of the Square of Opposition is that it allows a simple graphical representation of the *duality* relationship between quantifiers and, consequently, of the reciprocal "definability" between them. The square of opposition for standard quantifiers can be straightforwardly generalized for Generalized Quantifiers. A type 1 *generalized quantifier* over a domain E is a set $Q \subseteq \mathcal{P}(E)$. Denoting the *complement* of a quantifier Q over E by $\overline{Q} \stackrel{\text{def}}{=} \mathcal{P}(E) \setminus Q$ and the *postcomplement* as the set $Q- \stackrel{\text{def}}{=} \{A \subseteq E : E \setminus A \in Q\}$, the *dual* Q^d of a quantifier Q is the complement of the postcomplement: $\overline{(Q-)} \stackrel{\text{def}}{=} \{A \subseteq E : E \setminus A \notin Q\}$.

Thus complement, dual and postcomplement of a quantifier Q can be displayed along the edges of the square of opposition. Let Q be the generalized quantifier $Q = \forall = \neg\exists\neg = \{E\}$; the square for Q naturally unifies the squares for $\forall$ and $\exists$, representing the "extension" of the two quantifiers.

There is an important relation between quantifier duality and the so-called *scope dominance*. A quantifier Q_1 is *dominant* over Q_2 if the following unidirectional entailment is satisfied: $Q_1Q_2R \Rightarrow Q_2Q_1R$ (where Q_1Q_2R is an abbreviation of $Q_1xQ_2yR(x,y)$).

In the rest of the paper I will examine this relation and some of implications for the well-known phenomenon of scope ambiguity of natural language sentences with multiple quantification.

Reversed Squares of Opposition in PAL and DEL

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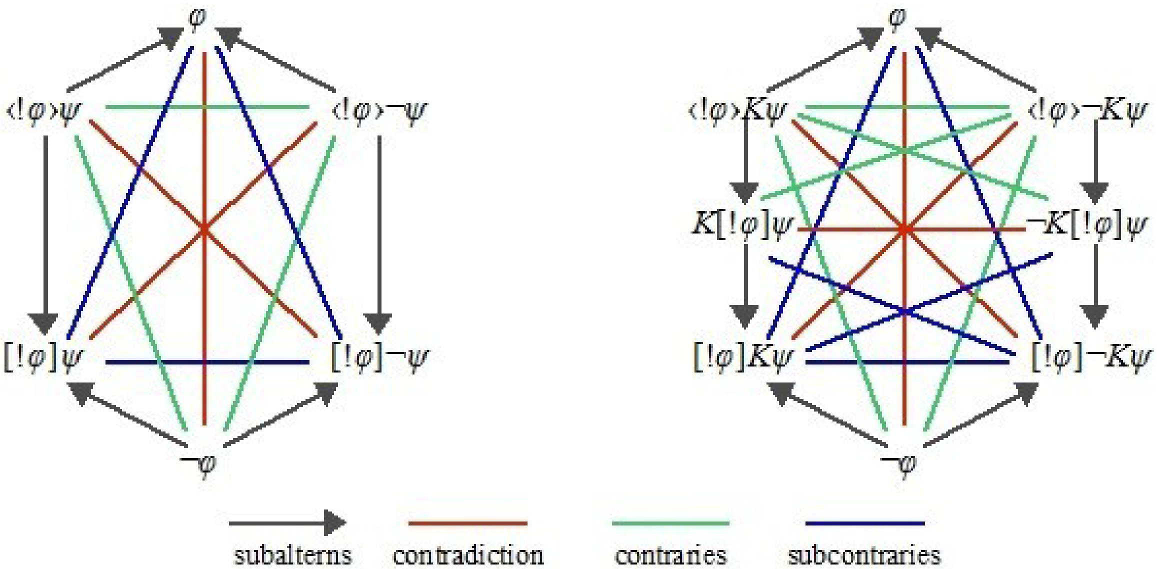
The main aim of this presentation is to show how nontrivial squares (and hexagons) of oppositions arise in public announcement logic (PAL) and dynamic epistemic logic (DEL). For ease of exposition, I will focus on PAL, a simple subsystem of DEL; however, all of the results presented have natural generalizations to DEL.

I will first introduce the technical apparatus of PAL, focusing on those aspects which will be of central importance from the square of opposition perspective. Considering the dual modal operators $[!\varphi]$ and $\langle !\varphi \rangle$ of PAL, it is tempting to construct a square of opposition with $[!\varphi]\psi$ and $[!\varphi]\neg\psi$ in the top corners, and $\langle !\varphi \rangle\psi$ and $\langle !\varphi \rangle\neg\psi$ in the bottom corners. However, this is not possible, because the partiality of public announcement leads to a violation of the subalternation relationships. I will then show that, because of the functionality of public announcement, we *do* get a 'reversed' square of opposition. Using standard techniques, this square can be extended to the left hexagon below.

Finally, I will consider the interaction between announcement and knowledge. Using PAL's expressive power, we can arrive at a very rich diagram (below on the right). This diagram represents in a compact way very much information about the subtle interactions between announcements and knowledge in PAL.



The Cube Generalizing Aristotle's Square in Logic of Determination of Objects (LDO)

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In this paper we present a generalization of Aristotle's square to a cube, in the framework of an extended quantification theory defined within the Logic of Determination of Objects (LDO).

The Aristotle's square is an image rending representational the link between quantifier operators and negation in the First Order Predicate Language (FOPL). However, the FOPL quantification is not sufficient to capture the « meaning » of all quantified expressions in natural languages. There are some expressions in natural languages which encode a quantification on **typical objects**. That is the reason for constructing a logic of objects with typical and atypical objects.

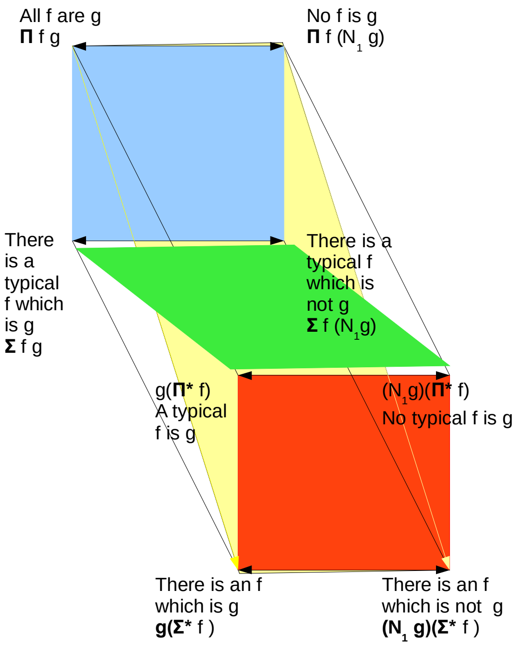
The Logic of Determination of Objects (LDO) (Desclés, 1986; Desclés and Pascu, 2007) is a new logic defined within the framework of combinatory logic (Curry, 1958) with functional types. LDO basically deals with two fundamental classes: a class of concepts (**F**) and a class of objects (**O**).

LDO captures two kinds of objects: **typical objects** and **atypical objects**. They are defined based on other primitive notions: the **intension of the concept (**Int(*f*)), the **essence of a concept *f*** (Ess(*f*)), the **determination operation** (δ), the **expanse of the concept** (Exp(*f*)), the **extension of the concept** f (Ext(*f*)).

Typical objects in Exp(*f*) inherit all concepts of Int(*f*); atypical objects in Exp(*f*) inherit only some concepts of Int(*f*).

LDO makes use of all the above notions and organizes them into a system which is a **logic of objects** (applicative typed system in Curry's sense (Curry 1958) with some specific operators). In LDO new quantifiers are introduced and studied. They are called **star quantifiers**: $\mathbf{\Pi}^{\square}$ and $\mathbf{\Sigma}^{\square}$ (Desclés and Guenthéva 2000). They have a connection with classical quantifiers. They are considered determiners of objects of Exp(*f*). They are different to the usual quantifiers **Π** and **Σ** expressed in the illative version of Curry (Curry, 1958) of Frege's quantifiers. They are defined inside the combinatory logical formalism (Curry 1958) starting from **Π** and **Σ**, by means of abstract operators of composition called combinators (Curry 1958). The system of four quantifiers **Π**, **Σ**, $\mathbf{\Pi}^{\square}$ and $\mathbf{\Sigma}^{\square}$captures the extended quantification that means quantification on typical/atypical objects.

The cube generalizing the Aristotle's square visualises the relations between quantifiers **Π**, **Σ**, $\mathbf{\Pi}^{\square}$ and $\mathbf{\Sigma}^{\square}$**.**



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The Square of Opposition in Orthomodular Logic

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In Aristotelian logic, categorical propositions are divided in: Universal Affirmative, Universal Negative, Particular Affirmative and Particular Negative. Possible relations between two of the mentioned type of propositions are encoded in the famous square of opposition. The square expresses the essential properties of monadic first order quantification. In an algebraic approach these properties can be represented taking into account monadic Boolean algebras [3]. More precisely, quantifiers are considered as modal operators acting on a Boolean algebra; the square of opposition is then represented by relations between certain terms of the language in which the algebraic structure is formulated. This representation is sometimes called the modal square of opposition. Several generalizations of the monadic first order logic can be obtained by changing the underlying Boolean structure by another one [4] giving rise to new possible interpretations of the square.

In this work, we consider the orthomodular logic enriched with a monadic quantifier and we provide interpretations of the square of opposition in several models of this logic as Boolean saturated orthomodular lattices [1], Baer*-semigroups and C^*-algebras [2].

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Pluralism in Logic: The Square of Opposition and Markoff's Principle

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The present study wants to represent by means of suitable kinds of SO the present pluralism in logic. I scrutinize the original scientific texts for recognising in which way the ancient and modern scholars made use of non-classical predicates. In each one of the texts presenting the theories of several scholars (Cusanus, Avogadro, Sadi Carnot, Lobachevsky, Kolmogoroff, Planck), a former part makes use of non-classical predicates, i.e. doubly negated predicates; this part concludes by means of a doubly negated predicate of an universal validity on the cases considered by the theory; the author, by changing this conclusion in the corresponding affirmative predicate, in the latter part considers it as an hypothesis from which he deductively develops in classical logic the theory. This complex pattern of arguing suggests as a first question which kind of SO one may suggest in non-classical logic. After solving such question I deal with a second question; i.e. to qualify in formal terms what justifies to change the kind of logic. I suggest that previous authors unwarily appealed to a principle which generalises that principle the constructivist Markoff surprisingly suggested in 1954 (this principle allows to change a doubly negated existential quantifier in the corresponding affirmative one, provided that the quantifier is a decidable one).

Soft Quantifiers in the Square of Opposition and Generalized Syllogisms

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The classical square of opposition ("every A is B", "no A is B", "some A is B", "some A is not B") may be softened into: for most x, $A(x) \rightarrow B(x)$; for most x, $A(x) \rightarrow \neg B(x)$; for few x, $A(x) \wedge B(x)$; for few x, $A(x) \wedge \neg B(x)$. Interestingly enough, this softening suppresses the con-tradictions along the diagonals. The systematic study of syllogisms (knowing that Q_1 A's are B's, Q'_1 B's are A's, Q_2 B's are C's, Q'_2 C's are B's, what can be said about A's that are C's, or C's that are A's?) involving quantifiers Q_i , Q'_j, such as 'most' and 'few', has been investigated almost two decades ago in artificial intelligence, but has remained largely ignored since during this time imprecise probabilities have not been much studied in reasoning under uncertainty [1]. In this work, 'most' and 'few' are respectively modeled by ill-known propor-tions supposed to belong to intervals respectively of the forms $[1-\alpha, 1)$ and $(0, \alpha]$, where α is an unspecified bound less than 1/2, while the interval $(\alpha, 1-\alpha)$ corresponds to a quantifier that can be understood as 'around half '. Then, using the optimal lower and upper bounds (not straightforward to compute) for Prob(C|A) that can be obtained from lower and upper bounds on Prob(B|A), Prob(A|B), and Prob(C|B), it has been possible to determine for what value of α, we preserve stable patterns of quantified syllogisms involving these vague quantifiers. The best value has been shown to be such that $\alpha \leq 1 - \sqrt{2}/2 \approx 0.29$, which guarantees that the value of Prob(C|A) remains in a given range such as $(0, \alpha]$, $(0, 1-\alpha]$, $[\alpha, 1)$, etc, when other proportions (probabilities) are in ranges corresponding to the ones representing 'most', 'few' or 'around half '. It is worth noticing that an understanding of 'most' as at least 70%, and of 'few' as less than 30% rather fits our intuition. Then, computations on probability bounds validate patterns such as if 'most A's are B's', 'most B's are A's', 'few C's are B's', 'most B's are C's', then '[aroundhalf , most] A's are C's', or if 'few A's are B's', 'most B's are A's', 'most C's are B's', 'few B's are C's', then 'few A's are C's', as well as a dozen of others, where most × most = [around half, most].

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Squares of Opposition in Generalized Formal Concept Analysis and Possibility Theory

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In formal concept analysis [2], a relation R is given between a set of objects Obj and a set of properties Prop. Let R(y) denote the set of objects having property y and $R^{-1}(x)$ the set of properties possessed by object x. Given a set Y of properties, four remarkable sets of objects can be defined in this setting [1]:

- $R^{\Pi}(Y) = \{x \in Obj \mid R(x) \cap Y \neq \varnothing\} = \cup_{y \in Y} R^{-1}(y)$, the set of objects having at least one property in Y;
- $R^{N}(Y) = \{x \in Obj \mid R(x) \subseteq Y\} = \cap_{y \notin Y} (R^{-1}(y))^{C}$, the set of objects having no property outside Y (where ^{C} denotes complementation);

- $R^{\Delta}(Y) = \{x \in Obj \mid R(x) \supseteq Y\} = \cap_{y\in Y} R^{-1}(y)$, the set of objects sharing all properties in Y (and having maybe some others). In fact, formal concept analysis has only made use of this set function which is enough for defining a formal concept as a pair made of its extension X and its intention Y such that $R^{\Delta}(Y) = X$ and $R^{-1\Delta}(X) = Y$, where $X \subseteq Obj$ and $Y \subseteq Prop$;
- $R^{\nabla}(Y) = \{x \in Obj \mid R(x) \cup Y \neq Obj\} = \cup_{y\notin Y} (R^{-1}(y))^{C}$, the set of objects that are missing at least one property outside Y.

The logical expressions underlying $R^{\Pi}(Y)$, $R^{N}(Y)$, $R^{\Delta}(Y)$, and $R^{\nabla}(Y)$ write respectively $\exists y, y\in R(x) \wedge y\in Y$; $\forall y, y\in R(x) \rightarrow y\in Y$; $\forall y, y\notin R(x) \rightarrow y\notin Y$; and $\exists y, y \notin R(x) \wedge y \notin Y$. Note that the two first expressions make a square of opposition together with the expressions corresponding respectively to $R^{N}(Y^{C})$ and $R^{\Pi}(Y^{C})$. Similarly, the two last expressions make another square of opposition together with the expressions corresponding to $R^{\Delta}(Y^{C})$ and $R^{\nabla}(Y^{C})$. However, the conditions $R^{\Pi}(Y)$, $R^{N}(Y)$, $R^{\Delta}(Y)$, and $R^{\nabla}(Y)$ make together a square of opposition of another type (where one goes both from R(x) to Y and from Y to R(x)).

Possibility theory provides a graded counterpart for this square (as well as for the two regular, previously mentioned, squares of opposition), see [1].

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Testing Two Competing Conceptual Tools: the Square of Opposition and Matrices of Concepts

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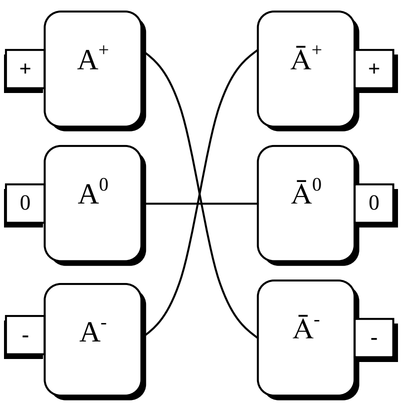
The "matrices of concepts" have been conceived of by the author as an alternative to the semiotic square. A matrix of concepts is built up from a pair of opposite concepts A/Ā. One can also consider that A and Ā are neutral concepts that can then be denoted by A^0 and $\bar{A}^0$. This leads to the construction of the class of canonical poles, when one considers an extension of the preceding class $\{A^0, \bar{A}^0\}$, such that A^0 and $\bar{A}^0$ admit respectively a positive and a negative correlative concept. The latter concepts can be denoted by $\{A^+, A^-\}$ et $\{\bar{A}^+, \bar{A}^-\}$. At this stage, for a given duality A/Ā, one gets the following concepts: $\{A^+, A^0, A^-, \bar{A}^+, \bar{A}^0, \bar{A}\}$, which constitute the *canonical poles*.



In the present paper, I draw a comparison between the square of opposition and matrices of concepts. I test then the construction resulting from both alternative conceptual tools with regard to three paradigm concepts: love, hate and indifference. I apply first the semiotic square to the love-hate-indifference triad. I encapsulate then the concepts resulting from the square of opposition into a framework which is best suited for comparisons. I make then use of matrices of concepts with the same triadic association of concepts. I also draw an accurate comparison between the two resulting series of concepts, that casts light on the common grounds of both conceptual tools. This also allows to point out accurately the differences between the square of opposition and matrices of concepts and suggests that the latter presents some advantages with regard to the square of opposition. Lastly, I extend the previous analysis to another paradigm pair of opposite concepts: masculine/feminine.

The Square of Opposition and the Vuillemin's Classification of Philosophical Systems

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In the book *What Are Philosophical Systems?* Jules Vuillemin has briefly considered the logical square of categorical sentences. Yet it turns out that the problem of categorical sentences as elementary sentences (in the Vuillemin's sense) is of greatest importance to his project. Let us remind that classification of philosophical systems by Vuillemin (*V-classification*, for short) is based on original assumptions. For example, the conviction that language moulds perception is deleted, since perception precedes language. Moreover, each form of predication becomes an ontological principle and results in exactly one philosophical system. Last but not least Vuillemin did not argue for some unique scheme of philosophical truth: he just made forward a suggestion about careful and serious consideration to the question what all possibilities of philosophical truth are. There is no easy answer to the problem. But we would like to ask if there are systematic (and/or erotetic) transitions ("switches") from the logical square of categorical sentences to the *V*-classification. The starting point is some part of the *inferential erotetic logic*.

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A hitherto Neglected Traditional Logical Square for Privations and Negations by Transposition

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In non-empty domains, sentences with different negation forms come to the same thing, as in the following examples:

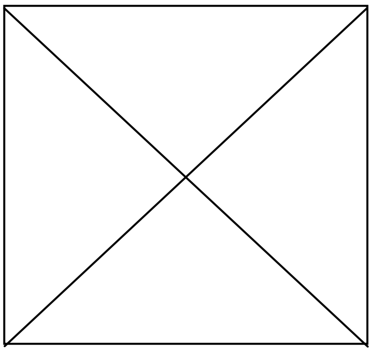
It is not the case that oppressors are just (weak negation, i.e. WN)
Oppressors are not just (negation by transposition, i.e. NT)
Oppressors are injust (privation, i.e. P)

But in empty domains or under some other circumstances, things can be rather complicated. In the following examples the different forms of negation result in different meanings:

It is not the case that zombies are just (WN*)
Zombies are not just (NT*)
Zombies are injust (P*)

P* says that zombies, being capable of it, are injust. NT* says either what P* says, or that zombies are not said to be just because they are not capable of being just or injust. WN* says either what NT* says, or that there are no zombies at all.

Affirmative	Zombies are just	Zombies are injust	P*
WN of NT*	It is not the case that zombies are not just	Zombies are not just	NT*
WN of P*	It is not the case that zombies are injust	It is not the case that zombies are just	WN*



On the right side, P* is stronger than NT* and this is stronger than WN*. On the left side, the affirmative is stronger than WN of NT* and this is stronger than WN of P*.

I will explore the above issues and present some traditional views towards a logic with three negations.

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A Dialectical Theory of Opposition

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Contrary to what is said in Aristotle's Posterior Analytics: *If a proposition is dialectical, it assumes either part indifferently; if it is demonstrative, it lays down one part to the definite exclusion of the other because that part is true*, I will claim there is another way to understand the oppositions of the dialectic. Firstly, by my interpretation of opposition, it will not be propositions, but functions that will be dialectical. Secondly, by denying the middle as Aristotle did in the Posterior Analytics; you have the problem of opposites being in a relation. For, if what is affirmed by the agent to be absolute, universal and or particular is true, its denial must be false. The conditional, also holds for the converse. Even the humble copula[1] is a way of relating terms, so the analysis can return to those conditionals, which avoid contradiction, or those that accept them. To accept the contradiction, either means that both the affirmation and the denial are taken to be true; else, they must both be false.

Categoricity and the Square of Opposition

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Smiley (1996) uses a result from Carnap (1943) to argue that standard, single-conclusion axiomatizations of classical propositional logic fail to capture the notions of contradictoriness, contrariness, and subcontrariness. The reason, he suggests, is that the axiomatizations allow for so-called non-standard valuations which do not respect the truth-conditional clauses for the logical connectives. Smiley calls this "a failure of categoricity at the sentential level". This notion of categoricity is made precise in Shoesmith & Smiley (1978). Smiley shows that categoricity can be restored either by using a multiple-conclusion system or by introducing signed formulae, +A and -A, indicating assertion and denial respectively. Schroeder-Heister (2007) has already connected the square of opposition to logics with explicit assertion/denial and proof-theoretic semantics. Here we suggest to connect that investigation with categoricity: How do we guarantee that our axiomatization has the expressive power to capture notions such as contrariness, subcontrariness, and contradictoriness in the semantics. We offer generalisations of Smiley's technique to give categoricity for finite many-valued logics. We then discuss the connection with the the square

[1] Hegel. G.W.F, 1830. Ibid. §214

of opposition, and ramifications for proof-theoretic semantics.

Aristotle's Square of Opposition: A Restricted Domain of Contradiction

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Aristotle's *On Interpretation* asserts four kinds of predication: a positive universal ('every man is just'), a privative universal ('no man is just'), a positive particular ('some man is just') and a privative particular ('not every man is just'). As for *Prior Analytics*, it introduces the relation of predication in the form of predicate-subject, as opposed to subject-predicate; e.g. 'A is predicated of every B' instead of 'every B is A'. This formal distinction *does* matter, as it shows that negation for Aristotle amounts to the privation of the subject in relation to a given predicate. Thus, 'A is not predicated of some B' corresponds to '*not* every B is A', which is not the same as 'some B is *not* A'. Now, the square of opposition can be explained by reference to the conversion rules of *Prior Analytics* (25a1-25). Aristotle tells us that 'every B is A' converts into 'some A is B', itself converting into 'some B is A'. By contrast, 'not every B is A' does not convert, as the predication is indeterminate; indeed, it could also mean 'some B is A'. In other words, 'A is predicated of some B' and 'A is *not* predicated of some B' can both be true at the same time, despite being opposite predications. This result violates the principle of non-contradiction, unless we grant the square of opposition, which claims that *contrary* opposites are distinct from *contradictory* opposites (*On Interpretation*, 17b17-26). While two contradictory opposites cannot both be simultaneously true, or simultaneously false, two contrary opposites can both be simultaneously false (if they are universal predications), or simultaneously true (if they are particular predications). By allowing contrary opposites to escape the principle of non-contradiction, Aristotle willingly restricts the domain of contradiction.

Mapping polyadic predicate logic into 12-bit code for a hypercube and tesseract of opposition

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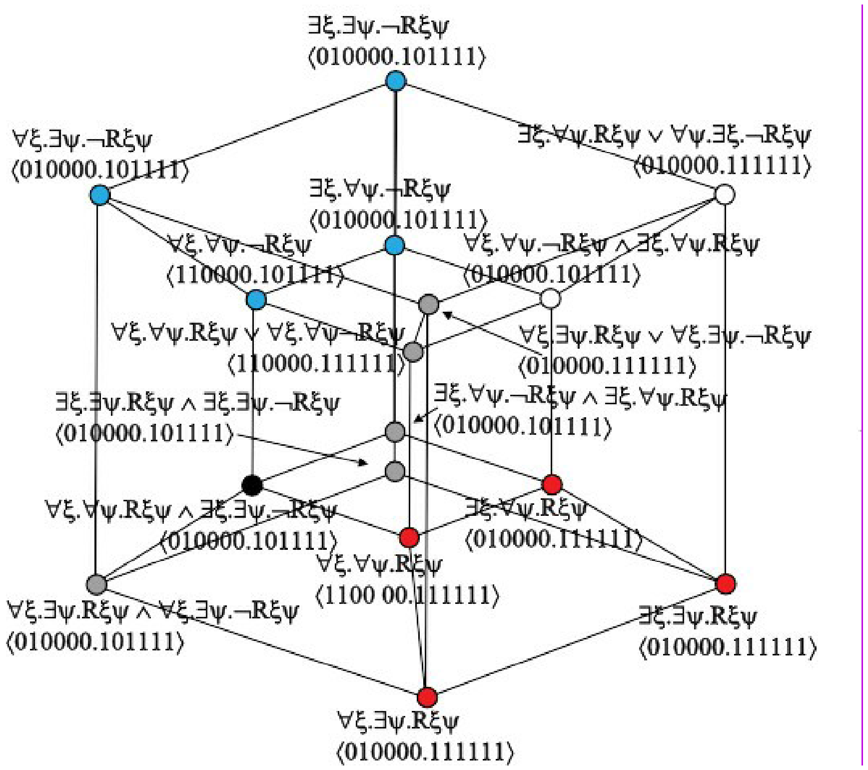
The quantifiers and verb formulas of polyadic predicate logic map into 12-bit codes that support up to four variables of { x, y, z, w}. Presented are 16 such codes for dyadic predicate logic in the two variables of {ξ, ψ} that form the vertices of a hypercube and tesseract of opposition. Results are that some squares of opposition in the hypercube may have vertices that are: diagonal of the same value or of opposite value; diagonal and adjacent of the same value; and all of unique value. This work was the result of abstracting the automated software tool named PETE for Polyadic Expander, Tokenizer, and Evaluator. The 8-bit code processed in PETE was expanded into the 12-bit code of PETE12. The tool is further abstracted into PETEn where the number of variables { n} determines the size of the bit code as the linear function (4 + 2*n) bits. What follows is that PETEn is an automaton that evaluates polyadic predicate logic as decidable.



General Patterns of Opposition Squares and 2n-gons

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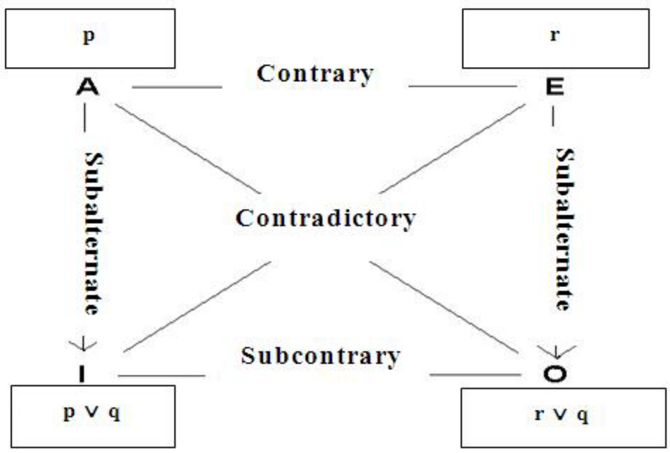
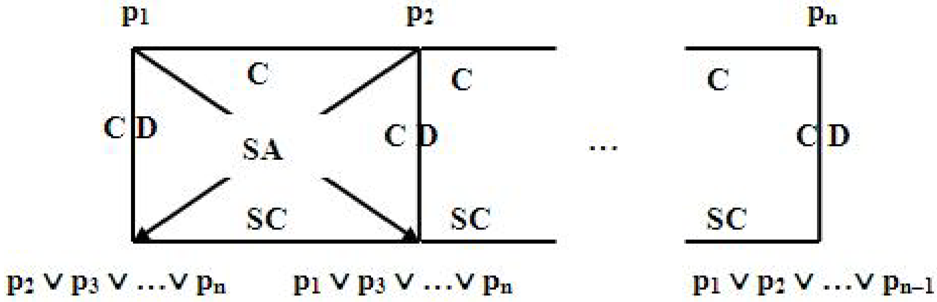
I will show that the classical square of opposition (SO) can be generalized to the "General Pattern of Squares of Opposition" (GPSO), which has two forms, denoted GPSO1 and GPSO2. For example, GPSO1 is as follows: if we have 3 propositions p, q, r that make up a trichotomy, then they can be used to construct the following SO:



However, the figure above shows an asymmetry among p, q and r: while each of p and r appears as independent propositions in the two upper corners, q only appears as parts of two disjunctions in the lower corners. To achieve symmetry, we need a hexagon composed of the following six propositions: p, q, r, $p \vee q$, $r \vee q$, $p \vee r$. Thus, a trichotomy is most naturally related to a hexagon rather than a square.

The relationship between a trichotomy and a hexagon may be generalized to the relationship between an n-chotomy and a 2n-gon. Given n propositions p_1, … p_n that make up an n-chotomy, we may construct the following 2n-gon of opposition (SA = subalternate, CD = contradictory, C = contrary, SC = subcontrary):



To achieve even greater generality, I will also explore the possibility of further generalizing the 2n-gons to 2^n-gons by generalizing the opposition relations.

Two Concepts of Opposition

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In systems of illocutionary logic, statements and features of statements are distinguished from illocutionary acts and features of illocutionary acts. Statements are understood to be true or false speech acts or language acts performed by uttering, writing, or thinking sentences, and are considered in abstraction from illocutionary force. Illocutionary acts of asserting/accepting, denying/rejecting, and supposing statements, among others, are what figure in actual deductive arguments constructed by human beings. These illocutionary acts are linked by rational commitment, so that performing some acts commits a person to performing others, and by coherence and incoherence. (It is incoherent to both assert and deny a single statement.) This paper explores the possibility of understanding elements of the (or a) square of opposition to be illocutionary acts, and of recognizing new forms of assertion and denial: universal assertion and universal denial, existential (particular) assertion and existential denial. The reconceived square can then be used to codify simple cases of commitment and incoherence.

Octaedron sive sphaera linguistica

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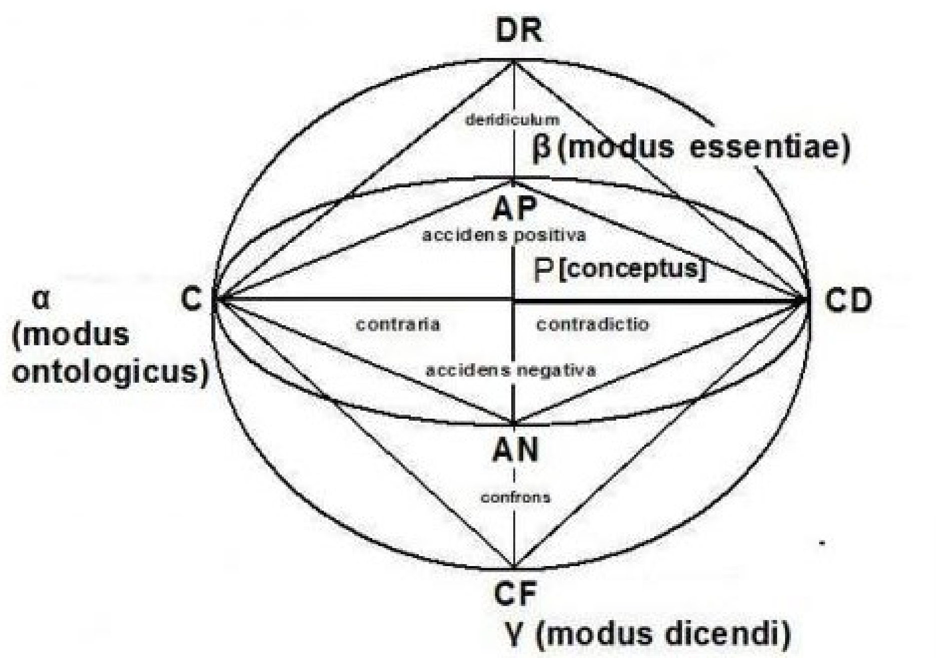

Both linguistics and philosophy of language inquire ordinary language by using their proper instruments and methods. Nevertheless the interdisciplinary spirit of Aristotle's *De Interpretatione* that deduces the rules of scientific language from the first principle of metaphysics (the principle of opposition), via logic and linguistics, seems to be rare nowadays. The present project concerns the logical understructure of ordinary language. Every language has besides ordinary concepts, which are considered having a neutral or a strict relation between a reference and a sign, the words, which are included as synonyms in the same families or sets, although in an actual learning process only the expertise of a native speaker, who explains the so called *relational meaning*, can teach adequate usage. The linguistic octahedron is an attempt to represent basic logical-linguistical relations between concepts with a neutral meaning or a strict reference and words that bear so-called *relational meaning.* The concept with a strict reference or a neutral meaning - *positio* - stands in the middle of the sphere. It is traversed by an α-onto-logical (*οντος λογία*) axis, β- ordinal axis and γ-modal axis. These axis are based on six oppositions and three triads. On the one hand the aim of the project was to offer an imaginative tool for displaying, organizing and mapping linguistical variety among specific entries in dictionaries like Thesaurus (concept, antonym, synonyms). The idea is to distinguish synonyms by their principal functions that could, in the end, serve as a useful visual instrument in learning process. On the other hand it seems that the basic axis of the figure reveals a lot of quasilogical relations (so called triangles of opposition) inside *depositing* circles.

Logical Oppositions and Collective Decisions

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We show how different ways of making collective decisions (for example, on the ground of majority, consensus, minority, veto) can be analyzed by a square of opposition where the quantification over a finite set of agents is represented. To that end, the square of opposition is extended to a positive and a negative lattice for *n* elements (agents) with ∀ and ∃

as the top and the bottom of an (inverted) Hasse diagram, respectively. In addition, we examine how different ways of making collective decisions affect the square of opposition of propositional and predicate logic. Some paradoxes of decision-making based on the majority vote are reproduced in the square, resulting in paraconsistent logics where non-contradiction and subalternation are not general rules of reasoning.

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How to Square Knowledge and Belief

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In his contribution to the 1st International Congress on "The Square of Opposition", Pascal Engel argued that the question "Can there be an Epistemic Square of Opposition?" probably should be answered in the negative. This scepticism, however, is unfounded. Taken for granted that knowledge satisfies the "truth-requirement" (saying that person *a* cannot know that *p* unless *p* in fact is true),

(T) $K(a,p) \rightarrow p$

it immediately follows that the *elementary* epistemic attitudes $K(a,p)$, $K(a,\neg p)$, together with their respective negations, fit into the following "square of knowledge":

$$\begin{array}{cc} K(a,p) & K(a,\neg p) \\ \\ \neg K(a,\neg p) & \neg K(a,p). \end{array}$$

As the traditional doctrine of the logical square wants it, (i) the two formulas on the upper line, $K(a,p)$ and $K(a,\neg p)$, are "contrary" to each other; (ii) the "subaltern" formulas on the lower line, $\neg K(a,\neg p)$ and $\neg K(a,p)$, are logically compatible; and (iii) the diagonally opposed formulas, i.e. $K(a,p)$ and $\neg K(a,p)$ on the one and $K(a,\neg p)$ and $\neg K(a,\neg p)$ on the other hand, are "contradictories" or negations of each other.

In contrast to $K(a,p)$, the operator of *belief*, $B(a,p)$ ("*a* believes that *p*), does *not* satisfy the truth-requirement. Even if 'belief' is interpreted in the strong sense of *certainty* (i.e. maximal subjective probability), it still doesn't follow that whenever subject *a* is firmly convinced that *p*, *p* must therefore be true – as we all know, humans are *fallible*. Nevertheless, if it is only assumed that the concept of ("rational") belief satisfies the weaker consistency principle

(C) $B(a,p) \rightarrow \neg B(a,\neg p)$,

one obtains the following "square of belief":

$$\begin{array}{cc} B(a,p) & B(a,\neg p) \\ \\ \neg B(a,\neg p) & \neg B(a,p). \end{array}$$

Furthermore, in view of the so-called "entailment-thesis" (which says that knowing that *p* entails believing that *p*),

(E) $\quad K(a,p) \rightarrow B(a,p)$,

both squares can be combined into the following "double square" (where each arrow symbolizes a logical implication):

$$\begin{tikzcd}
K(a,p) \arrow[dr] & & & K(a,\neg p) \arrow[dl] \\
& B(a,p) \arrow[d] & B(a,\neg p) \arrow[d] & \\
& \neg B(a,\neg p) \arrow[dl] & \neg B(a,p) \arrow[dr] & \\
\neg K(a,\neg p) & & & \neg K(a,p)
\end{tikzcd}$$

Interestingly, given certain assumptions of epistemic/doxastic logic, the doxastic "inner square" may be shown to be *identical* to the following square of *complex* epistemic modalities:

$$\begin{tikzcd}
\neg K(a,\neg K(a,p)) \arrow[d] & \neg K(a,\neg K(a,\neg p)) \arrow[d] \\
K(a,\neg K(a,\neg p)) & K(a,\neg K(a,p))
\end{tikzcd}$$

Aristotle and Galois

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Evariste Galois is known to be one of the founders of group theory. His work on algebraic solutions of polynomial equations also laid the foundations of what is called Galois theory. This was the motivating example of the abstract notion of Galois connection, a special kind of correspondence between posets that pervades mathematics. In this talk we will give a logical characterization of Galois connections between powersets, which encompass most of the key examples. More precisely, we are going to show that the underlying logical structure is essentially a combination of two copies of the square of oppositions. On the way we will give a rudimentary `geometricological' analysis of this latter, in the spirit of group theory, to argue that its underlying geometrical structure is actually not a square. No prerequisite will be assumed except some familiarities with ordered sets and basic group theory.

A Square of Opposition of Moral-Legal Evaluation-Functions in Two-Valued Algebra of Formal Ethics

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A square of opposition of moral-legal evaluation functions in formal ethics is a *generalization* of the square of opposition in formal logic. As thinking is a particular case (kind) of human acting, the algebraic system of thoughts is a subsystem of an algebraic system of acts. The "true" is a particular case of the "good". The "false" is a particular case of the "bad". Algebra of formal ethics is based upon the set of acts. By definition, acts are such and only such operations, which are either good, or bad ones. Algebraic operations defined on the set of acts are moral-legal evaluation functions. Variables of these functions take their values from the set *{g,b}*. The functions take their values from the same set. Symbols *g* and *b* stand for moral-legal values of acts "good" and "bad", respectively; symbols *x* and *y* – moral-legal forms of acts. Elementary moral-legal act forms – independent variables. Compound moral-legal act forms – moral-legal evaluation functions of these variables. In the two-valued algebra there are 16 mathematically different binary operations. *Formal-ethical quantifiers* considered as *binary* moral-legal evaluation-functions are among the 16. The moral-legal quantifiers are defined by means of the following glossary and evaluation-table.

Glossary for the table 1: The symbol *Uxy* stands for the moral-legal evaluation-function "*universality* of *y* for *x* ". [In other words, *Uxy* means a moral-legal act of *y*'s being universal for *x*.] The symbol *PxNy* stands for moral-legal evaluation-function "*particularity* (i.e. not-universality) of *not-y* for *x* ". *UxNy* – moral-legal evaluation-function "*universality* (i.e. not-particularity) of *not-y* for *x* ". *Pxy* – "*particularity* (i.e. not-universality) of *y* for *x* ". [In other words, *Pxy* is a moral-legal act of *y*'s being particular (not universal) for *x*.] The evaluation-functional sense of these binary operations is defined by the table 1.

Table 1. Quantifiers as moral-legal evaluation-functions determined by *two* variables

x	*y*	*Uxy*	*PxNy*	*UxNy*	*Pxy*
g	g	b	g	b	g
g	b	b	g	b	g
b	g	g	g	b	b
b	b	b	b	g	g

Glossary for the below evaluation-table 2: The symbol U^1y stands for the moral-legal evaluation-function "*universality* (generality) *of y*". The symbol P^1y means the moral-legal evaluation-function "*particularity* (i.e. not-universality) *of y*". *Ny* stands for the *unary* moral-legal operation "*negation* (destruction, termination, annihilation) *of y*". *Oy* – the *unary* moral-legal operation "*opposition to y* (or *opposite of y*)". The evaluation-functional sense of these *unary* operations is precisely defined by the following table 2.

Table 2. Quantifiers as moral-legal evaluation-functions determined by *one* variable

y	U^1y	P^1y	*Ny*	*Oy*
g	g	b	b	b
b	b	g	g	g

Let the symbol «*x=+=y*» stand for the relation: «moral-legal evaluation-function *x* is *formally-ethically equivalent* to moral-legal evaluation-function *y*». In the algebra of formal ethics, *a moral-legal evaluation-function x is called formally-ethically equivalent to a moral-legal evaluation-function y, if and only if these functions (x and y) acquire identical moral-legal values (g or b) under any possible combination of moral-legal values of the variables (of these functions).* Using the above definitions, it is easy to obtain the following equations representing the *formal-ethical square of opposition*. 1) *Uxy=+=OPxy*. 2) *Pxy=+=OUxy*. 3) *UxNy=+=OPxNy*. 4) *PxNy=+=OUxNy*. According to these equations; universality is (formally-ethically) opposite to particularity; particularity is (formally-ethically) opposite to universality.

Consequences of an Extensional Viewpoint for the Lexical Interpretation of the Logical Square

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Notwithstanding reciprocal divergences, the many theoretical proposals that have contributed to the debates on the logical square throughout its history normally present one constant characteristic: the predominance of intentional definitions over extensional ones. This paper aims at the study of the possibilities generated by extensional interpretations of this structure by means of the lexical manifestations of the oppositions in the square.

An extensional perspective may pave the way for new inferential possibilities, since it considers opposing lexical items as denoting objects placed in ordered regions. In this research, we will investigate the logical bases for the passage from the traditional structure of opposition to this new form of opposition, providing examples from applications on specific semantic fields in natural language. We will show that this form of logical square implies the adoption of gradual oppositions (instead of discreet and privative oppositions) and of an asymmetric ordination of the opposing terms.

A Logical Framework for Debating the Scientific and Social Implications of Modern Technologies

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By providing concepts and techniques that abide by rigorous schemes, science sets the stage for the development of possible technologies that can deeply change our daily lives. But it has become more and more essential that moral, ecological, political and legislative considerations be taken into account to assess all implications in the deployment of new technologies, in particular when they can have strong negative effects and jeopardize the well being of current or future generations.

The logic we propose aims at facilitating decision making, as the main outcome of debates involving local communities and scientists on issues related to the use of technologies and their potential impact on the environment, health, and culture.

We base our framework on a variant of the S5 epistemic logic that accepts 3 pairs of modalities (of possibility and necessity): it allows one to qualify not only the property of being true, but also the property of being practical and the property of being useful.

Opposition from the viewpoint of Universal Logic

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Negation, denial and refutation are fascinating many-sided operations, and the Aristotelian Square of Oppositions (SoO) is a well-rounded diagrammatic endeavor to capture some of their main facets and interrelationships, for the case of the so-called categorical propositions. The multiple ways in which a given statement-form can be countered by another statement-form are usually characterized semantically, with the help of the two classical logical values, true and false: for a given *n*-ary statement ©, its *contradictory* ×© is such that © and ×© always assume different truth-values, its *subalternate*

$\Downarrow$© is such that it inherits the value of © in case this value is true, its *contrary* $\cap$© is such that © and $\cap$© cannot both be true (but may both be false), its *subcontrary* $\cup$© is such that © and $\cup$© cannot both be false (but may both be true). Among these operations, subalternation is the only non-involutive one. It raises in fact several difficult technical issues in the theory of quantification, and does not seem anyway to convey any reasonable sense of *opposition*, to start with. My first proposal here, thus, is to exchange $\Downarrow$ by a more general and involutive *duality* operator ◊ that does offer a useful perspective on opposition. Next, I show that contradictoriness and duality may both be characterized from the point of view of Universal Logic, using nothing but the abstract theory of (symmetric) consequence relations, and a side-effect of this is that a genuine proof-theoretical approach to the SoO is ready at hand. A suitable combination of × and ◊, as applied to a given node comprising a statement-form ©, may obviously be used to generate the other nodes of a related SoO. Furthermore, the mentioned universal approach lifts in a wholly natural way from classical propositional logic to quantificational, modal or many-valued logic, or to any other tarskian/scottian logic, irrespective of its circumstantial semantical characterization. It's not all a geometrical bed of roses for the universal approach, of course. As it should be expected, some statement-forms simply do not have contrary or subcontrary counterparts, and a characterization of the initial semantical ideas behind $\cap$ and $\cup$ from the point of view of Universal Logic is found wanting: for the well-studied unary and the binary cases, the troubles appear when a given statement-form is self-dual, or when its dual counterpart is identical to its contradictory counterpart, as such situations make the square collapse into a line; however, in the analysis of the generic *n*-ary case, as I will show, the situation gets more involved.

The Rationalist Response to the Theological Disquietude Caused by the Application of Subalternation to Essential Definitions

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Aristotle bequeathed two views on essential definitions: such definitions are necessary universal affirmatives, and universal affirmatives entail particular affirmatives. Though Neoplatonic logicians like Proclus adapted both to syllogistic descriptions of the necessary emanation, they cause problems for Christian philosophers who held to a correspondence theory of truth, divine omniscience, and God's free creation of the world in time. How could an omniscient God know prior to creation the truth of *every man is rational* when there was nothing for its terms to correspond to? How could the creation of man be free if it is necessary that he be rational, and this necessary truth logically entails what would have to be a necessary existential proposition? This paper explores various attempts to reconcile these views culminating with "success" only in the later Rationalists. What became the standard medieval account, which posits of a second "timeless" sense of the copula that corresponds to "objective beings" in God's mind, sacrifices subalternation of an I proposition with existential import. Though at times Descartes seems to accept the medieval view, it clashes with his explanation of the source of human error: belief in universal affirmatives propositions with false ideas as subject terms. If a universal affirmative is false because its subject term is empty, subalternation must be valid. The clash is aggravated in Arnauld's development of Cartesian logic by his definition of truth in terms of extension. If a subject term is empty, then its extension is identical to the extension of the predicate restricted by that of the subject because both extensions are empty. Hence if true universal affirmatives did not exclude false subject terms, all universal affirmatives with false idea as subject would, on Arnauld's

account, be automatically true. Malebranche accepts the other horn of the dilemma, identifying true universal affirmatives as necessary identities between God's ideas that do not necessarily entail that they be exemplified among things in the world. Aristotle's logical views are reconciled with the Christian doctrines only with Spinoza and Leibniz, who eliminate the correspondence problem for truths prior to creation by denying that substances have a beginning in time, and eliminate the clash between God' freedom and necessary existentials by distinguishing new senses of "freedom" compatible with the logical facts. By his relational theory of time, moreover, Leibniz explains how one contingent possible world might be earlier than another though each exemplifies necessary essences.

The Klein Four-Group and Squares of Opposition in Categorical Syllogistic

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A group, (G, ∘), is a mathematical structure in abstract algebra consisting of both a set of elements, G, and a binary operator, ∘, closed and associative over the set G which also includes an identity element and for every member, an inverse element. Categorical syllogistic is replete with logical structures informed by a special group, K_4, called the Klein four-group. These structures are the results of the group action of K_4 groups of four distinct, but related sets of four unary operators acting 1.) on categorical propositions of one term (A_P; I_P; E_P; O_P), 2.) on categorical propositions of two terms (A_{SP}; I_{SP}; E_{SP}; O_{SP}), 3.) on categorical arguments of three terms, in particular, Aristotlean syllogisms in perfect mood ($A_{MP}A_{SM}A_{SP}$; $A_{MP}I_{SM}I_{SP}$; $E_{MP}A_{SM}E_{SP}$; $E_{MP}I_{SM}O_{SP}$), and 4.) on categorical arguments of four terms, in particular, Peircean arguments from analogy in the perfect mood ($A_{MP}A_{MN}A_{SN}A_{SP}$; $A_{MP}A_{MN}I_{SN}I_{SP}$; $E_{MP}A_{MN}A_{SN}E_{SP}$; $E_{MP}A_{MN}I_{SN}O_{SP}$). These group actions yield interrelated squares of logical opposition. This essay defines the particular characteristics of these various K_4 groups and their respective group actions on sets of categorical schemata of one, two, three, and four terms. The resulting squares of logical opposition will be diagrammed and then explicated using the concrete K_4 groups of D_2, $S_2 \times S_2$, and $\mathbb{Z}_2 \times \mathbb{Z}_2$. Furthermore, it will be demonstrated that these squares of opposition are also instances of Pythagorean *analogia*, that is, four-place analogies of the form A:B::C:D. Finally, a hypercube of logical opposition will diagram the major central discoveries of this presentation. In general, this essay reveals the role group theory can play in depicting the mathematical structure of syllogistic.

One and a Half Square of Opposition in Linear Logic

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Linear logic was created by Jean-Yves Girard in 1987. It allows to make a distinction respectively between two kinds of conjunction and two kinds of disjunction, one of each being called "multiplicative" and the other ones "additive." One of this logic's main properties is to combine the symmetry of classical logic with the constructivity of intuitionistic logic.

It will be examined whether logical squares may be built in linear logic. As we will see, the multiplicative fragment offers a perfect square of opposition, whereas the additive one does not. Nevertheless, against some expectations, the latter incomplete square is "not as incomplete" as its intuitionistic analogon. Thus linear logic delivers perhaps the maximal symmetry that constructivity may suffer.

No previous knowledge of linear logic is assumed: its principles will be described during the conference.

A Cube Extending Piaget's and Gottschalk's Formal Square

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"*N*-Opposition Theory" (NOT) is a new branch of mathematics, at the intersection of logic and geometry (similar to "graph theory" and "knot theory"). It generalises the logical "square", "hexagon" and "cube": they are the first three terms of an infinite series of "logical bi-simplexes of dimension *m*". By means of a game-theoretical ask-answer device, the "Aristotelian p^q-semantics" (generating correlative "Aristotelian p^q-lattices") NOT develops "logical poly-simplexes of dimension *m*" (a generalisation of the logical bi-simplexes). In this paper we show how, starting from Piaget's "INRC square" and Gottschalk's "square of quaternality", mutually isomorphic (henceforth the "PG-square") – a square differing interestingly from Aristotle-Apuleius' one –, we can, by a suited game-theoretical "Piagetian-Gottschalkian p^q-semantics", develop "avatars" of the PG-square similar, *mutatis mutandis*, to the NOT-theoretical avatars of the logical square. Here we will study the first of them, the "PG-cube". More generally, this suggests that there is a whole "logical PG-geometry", parallel to the rich one of NOT. Future will tell whether both are instances of a still more abstract underlying game-theoretical "logical geometry".

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A Diagrammatic Calculus of Syllogisms

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Our aim is that of introducing and discussing a diagrammatic calculus of syllogisms on the base of which a syllogism is valid if and only if its conclusion follows from its premisses by calculation. In order to understand and use the calculus, no particular knowledge or ability is required, so that it could be said to be algorithmic, in a naive sense. The calculus permits to express the relations of contradiction, subalternation, contrariety and subcontrariety that hold between the categorical propositions in the square of opposition. Furthermore, it permits the easy deduction of the traditional rules of syllogism as well as the direct calculation of the well-known formula for the number of valid syllogisms. All the above can be carried out informally, on the base of some intrinsic features of the calculus itself but, in order to rigorously justify it, a suitable mathematical framework has to be singled out. This is done by pointing out the existing connections with the theory of rewriting systems, through the adoption of a polygraphic approach. Eventually, the looked for mathematical framework will turn out to be in fact category theoretic.

Constructive Results in the Logic of Categorical Propositions

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In 1933 Gödel proved that we cannot distinguish classical logic from intuitionistic logic with respect to their theorems in the fragment $\{\wedge,\neg\}$. Although the fragments $\{\forall,\neg,\wedge\}$ and $\{\exists,\wedge,\neg\}$ are sufficient to establish a distinction between classical logic and intuitionistic logic, there are several constructive results that can be proved in these fragments. For example, we can prove that negation is constructively involutive in the fragment $\{\forall,\neg,\wedge\}$ and that every classical theorem of the form $\exists xA(x)$ with $A(x)$ quantifier free is intuitionistically provable in the fragment $\{\exists,\wedge,\neg\}$. The aim of the present paper is to show some constructive consequences for the logic of categorical propositions and for the the logic of modal propositions. We shall show in particular that the categorical square of oppositions and the modal square of oppositions are completely constructive with respect to theorems.

An Elementary Square of Opposition at the Basis of Analogical and Other Related Proportions

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The Aristotelian square of opposition (between four logically related statements) is in some sense at the root of syllogistic and deductive reasoning. In Greek-originated terminology, there is a basic distinction between top-down forms of reasoning aiming at cataloguing items by classifying them into categories and subcategories, and down-top forms aiming at analogizing, i.e., comparing particular items. Recent attempts at formalizing analogical proportion (i.e., a statement of the form « a is to b as c is to d »), have led to define it by the logical formula $(a\wedge\neg b \equiv c\wedge\neg d) \wedge (\neg a\wedge b \equiv \neg c\wedge d)$, which expresses that a differs from b (resp. b differs from a) just as c differs from d (resp d differs from c), see e.g. [1]. A related proportion, named 'paralogy' that expresses that what a and b have in common, c and d have it also, rather equalizes the similarities between a and b with the similarities between c and d, and is defined by $(a\wedge b \equiv c\wedge d) \wedge (\neg a\wedge\neg b \equiv \neg c\wedge\neg d)$. Clearly, when comparing two situations described by the sets of properties that hold for them, their similarity pertains to what they have in common positively or negatively, while their dissimilarity refers to the sets of properties that one has and the other has not. At the logical level, similarity thus corresponds to the conjunctions $s1= a\wedge b$ and $s2 = \neg a\wedge\neg b$, while dissimilarity corresponds to $d1 = a\wedge\neg b$ and $d2 = \neg a\wedge b$. These four expressions can be arranged into an elementary square of opposition, where one moves horizontally (resp. vertically) by negating a (resp. b) (no quantifiers appear in it since analogizing only involve instantiated situations).

Analogy $(d1\equiv d'1 \wedge d2\equiv d'2)$ and paralogy $(s1\equiv s'1 \wedge s2\equiv s'2)$ give themselves birth to another square of opposition where two other related proportions appear (the primed symbols refer to c and d): inverse analogy $(d1\equiv d'2 \wedge d2\equiv d'1)$ and reverse paralogy $(s1\equiv s'2 \wedge s2\equiv s'1)$. There are many other logical proportions that can be defined through two equalities relating two of the 4 expressions pertaining to (a, b) with two of the 4 ones pertaining to (c, d). Among them let us mention conditional proportions defined by expressions such as $(s1\equiv s'1 \wedge d1\equiv d'1)$, which corresponds to the equivalence between the two 'conditional objects' $b|a$ and $d|c$, expressing that they have the same examples and the same counter-examples [1]. The 16 conditional proportions can be also organized in squares of opposition.

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Approaches in Computability Theory and the Four Syllogistic Figures

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What about "recipe-driveness" as a link between the venerable square and computability theory, i.e. leading to the discovery, understanding and classification of decidable /undecidable problems? — Here, we are especially interested in the question of consistency as the shibboleth of trustworthiness. At least since Behmann's paper 1931 [1] the source of paradoxes resp. inconsistencies is located in the definitional machinery. Behmann suggested a revised type-free logic with an additional operator *!* which, given a predicate χ, singles out exactly those arguments x to which χ meaningfully applies. Then, for instance, the syllogism Barbara, usually stated in the form

$(\Box x)(A(x) \to B(x)) \Box (\Box x)(B(x) \to C(x)) \to (\Box x)(A(x) \to C(x)),$

is modified to

$(\Box x)(A(x) \to B(x)) \Box (\Box x)(B(x) \to C(x)) \to (\Box x)(C(x))! \to (A(x) \to C(x))),$

where the range of the last quantifier is restricted to a *!*-sample of x for which it does make sense. We discuss Behmann's not laboured theory, give a proposal how to interpret his special operator *!* and compare this with work of Aczel and Feferman also inspired by Behmann [2]. —
But, “the ghost of the Tarski hierarchy is still with us”[3]?

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Towards a New View of Dialectical Synthesis that does not attempt to Round the Square

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Those who respect binary logic avoid the terminology of ‘dialectical synthesis’ because they want to avoid the anti-binary logic version of the concept developed by Hegel and Marx. We need to reclaim the concept for the understanding and practice of the rational discussion of alternatives, but rid it of the anti-binary logic elements. According to Hegel and Marx, all opposites are contradictory; the resolution of the contradiction in a synthesis shows that the opposites have elements of the truth which are united in the synthesis—a mid point that denies the excluded middle. Contrary to Hegel and Marx, a synthesis takes contrary (not contradictory) and at times complimentary alternatives and melds them into a new viewpoint that corrects and improves on the alternatives: forming a new corner on a new square. Also, Hegel wanted to correct the mistaken idea that if we use the same words then the meanings or concepts must be the same. But, concepts and meanings change with the development of knowledge despite a conservation of the terminology. Hegel goes overboard in arguing that the 'law of identity' is wrong because it does not account for changes in concepts and meanings. Actually, the reverse of what Hegel says about the 'law of identity' is true: we are reminded by the 'law of identity' to examine the meaning not the words. The upshot is that contrary to Hegel and Marx, dialectical synthesis need not round the square of opposition by denying binary logic. Thanks Aristotle.

Mulla Sadra on the Conditions of Contradiction

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According to the Aristotelian logic, the relation of contradiction, as the heart of the square of opposition, lies between a proposition and its negation so that necessarily one of them is true and the other false. However, it seems that the mere difference in affirmation and negation is not a sufficient condition since "John is sick" and "John is not sick" may be both true if two different periods of time is meant. Aristotle himself seems to be aware of this fact when he talked of "any further qualifications which might be added" during his attempt to characterize the so-called "principle of contradiction" in his *Metaphysics*. (1005b) After Aristotle Muslim logicians and philosophers have being tried to disclose these "further qualifications" in several ways. According to the most detailed dictum it was said that two contradictory quantified (non-modal) propositions must be different in respect of *quantit*y (i.e., being universal or particular) and *quality* (i.e., being affirmative or negative) and similar in respect of *subject, predicate, time, place, condition, relation, actuality/potentiality* and *whole/part*. The second part of this dictum has been known as "the eight similarities" (*al-wahadat al-thamaniah*). Mulla Sadra, however, added a ninth similarity as a necessary condition of contradiction; i.e., similarity in respect of *predication*. This condition is based on Sadra's distinction between two kinds of predication: the essential (*al-dhatee*) and the common (*al*-shayea). In this paper I shall first examine this distinction and then try to compare it with some modern distinctions such as "analytic/synthetic" and "mention/use". Eventually I shall investigate its role as a condition of contradiction.

Truth Tables and Oppositional Solids

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The truth table method is long familiar to logicians. It is used to calculate the values of a truth function, i.e. a function from a complete set of logical configurations to the set {true, false}. As shown in Sart (2009), the truth table method can be extended to deontic logic. In that case, a logical configuration is no longer an alethic assignment, i.e. an assignment of alethic values (true or false) to base propositions, but a more complex object I call an "alethico-deontic assignment".

Given a set N of n propositions, it can be shown that the alethico-deontic space $E_1(N)$, i.e. the set of all alethico-deontic assignments based on N, has $2^n \times 2^{2^n}$ elements. From this it follows that the number of truth functions on the alethico-deontic space $E_1(N)$ is $2^{2^n \times 2^{2^n}}$. In the case where $n = 1$, the simplest case, there are thus 8 alethico-deontic assignments and $2^8 = 256$ truth functions.

Let $E_D(\{p\})$ be the subspace obtained by removing from the alethico-deontic space $E_1(\{p\})$ the two "evil" configurations (those in which everything is forbidden). I shall show that each of the 62 non-trivial truth functions on the subspace $E_D(\{p\})$ is determined by one and only one of the 62 formulas decorating the 5-dimensional hyper-tetraicosahedron given in Moretti (2009), Section 6. In other words, I shall show that Moretti's 5-dimensional hyper-tetraicosahedron is a faithful representation of KD45($\{p\}$), the modal logic behind the subspace $E_D(\{p\})$.

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A Theory of Opposites

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An opposition OP is the relation R that obtains between two relata *a* and *b* (concepts or sentences), while an opposite is the result of the function *op* that turns a relatum into the other one. Thus if *b* is this result, then *op*(a) = b, and OP(a,*op*(a)) stands for the whole relation of opposition between *a* and *b*. We give some illustrations of OP and *op* in two special cases: the opposition of classical connectives, and the opposition of modalities. It will be argued that: the theory of opposites finds its roots in the seminal works about quaternality by Piaget (1972) and Gottschalk (1953), mentioned by Blanché (1966) but whose peculiarity hadn't been noted by him; the theory of opposites is structurally prior to the theory of opposition: no relation OP can be formed without *op*; each case of opposite-forming operator *op* corresponds to a unary operator, whereas OP corresponds to a family of binary connectives. We review a sample of such unary operators: they form a variety of logical negations, but their specific properties remain to be identified beyond the preliminary insights of Béziau (2003). We'll do this by means of an algebraic framework: a Question Answer Semantics (shorthand: **QAS**), which helps to characterize the terms of an opposition and their transformations by means of logical values.

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Syllogistic Structure for Symbolic Representation of Information

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Information is understood here in the tradition of the philosophical theme of the one-many relationship, as that which makes a variety (carrier of information) one, for instance by a selection one out of the many (selective information), or through the structure which binds many components into one (structural information). Using this definition a mathematical model has been proposed by the author in the past for information and its processing, including information integration occurring for instance in the process underlying phenomenal characteristics of consciousness.

The present paper is an attempt to approach the task of modeling semantic information by introducing the concept of a symbol as a representation of the carrier of a large volume of information by that of a small volume. Thus, symbol does not represent an object of different ontological status as a carrier of information pointing at it, but it is a relation between two carriers of information of usually very different volumes. The main difference between the present approach and the only earlier attempt by Bar-Hillel and Carnap is that while they were developing their theory of semantic information on pre-existing logical structure, in this paper the mathematical structure modeling information (closure space) is a point of departure, and the logical structure of the type of syllogistic is reconstructed within this model. In this context, the square of opposition appears as a link between the relations defining the model of syllogistic and their alternatives of special importance for the study of information.

Non-Archimedean Explanation of the Square of Opposition

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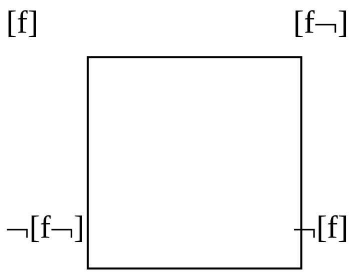
Suppose B is a complete Boolean algebra with the bottom element 0 and the top element 1 such that the cardinality of its domain $|B|$ is an infinite number. Build up the set B^B of all functions $f : B \to B$. The set of all complements for finite subsets of B is a filter and it is called a Frechet filter, it is denoted by U. Further, define a new relation $\approx$ on the set B^B by $f \approx g = \{a \in B : f(a) = g(a)\} \in U$. It is easily proved that the relation $\approx$ is an equivalence. For each $f \in B^B$ let $[f]$ denote the equivalence class of f under $\approx$. The ultrapower B^B/U is then defined to be the set of all equivalence classes $[f]$ as f ranges over B^B. It is denoted by $^*B$.

There exist two groups of members of $^*B$: (1) functions that are constant, e.g. $f(a) = m \in B$ on the set U, a constant function $[f = m]$ is denoted by $^*m$, (2) functions that aren't constant. The set of all constant functions of $^*B$ is called standard set and it is denoted by $°B$. The members of $°B$ are called standard. It is readily seen that B and $°B$ are isomorphic.
We can extend the usual partial order structure on B to a partial order structure on $°B$: (1) for any members $x, y \in B$ we have $x \leq y$ in B iff $^*x \leq {}^*y$ in $°B$, (2) each member $^*x \in °B$ (which possibly is a bottom element ^{*0} of $°B$) is greater than any number $[f] \in {}^*B \backslash °B$, i.e. $^*x > [f]$ for any $x \in B$, where $[f]$ isn't constant function. Notice that under these conditions, there exist the top element $^*1 \in {}^*B$ such that $1 \in B$, but the element $^*0 \in {}^*B$ such that $0 \in B$ is not bottom for $^*B$. Introduce three operations 'sup', 'inf', '$\neg$' in the partial order structure of $^*B$: $\inf([f], [g]) = [\inf(f, g)]$; $\sup([f], [g]) = [\sup(f, g)]$; $\neg[f] = [\neg f]$. Consider the member $[h]$ of $^*B$ such that $\{a \in B: h(a) = f(\neg a)\} \in U$. Denote $[h]$ by $[f\neg]$. Then we see that $\inf([f], [f\neg]) \geq {}^*0$ and $\sup([f], [f\neg]) \leq {}^*1$. Really, we have three cases.

- *Case 1*. The members $\neg[f]$ and $[f\neg]$ are incompatible. Then $\inf([f], [f\neg]) \geq {}^*0$ and $\sup([f], [f\neg]) \leq {}^*1$,
- *Case 2*. Suppose $\neg[f] \geq [f\neg]$. In this case $\inf([f], [f\neg]) = {}^*0$ And $\sup([f], [f\neg]) \leq {}^*1$.
- *Case 3*. Suppose $\neg[f] \leq [f\neg]$. In this case $\inf([f], [f\neg]) \geq {}^*0$ And $\sup([f], [f\neg]) = {}^*1$.

As a result, we obtain the square of opposition, e.g. in case $[f\neg] \leq \neg[f]$ we have:



Does a leaking O-corner save the Square?

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Some late 20th-century American philosophers have tried to save the Square by taking away existential import (EI) from the **E**- and **O**-corners ('leaking the **O**-corner'). When this is done, the Square survives intact: all its logical relations are preserved, but without undue EI. (Aristotle, followed by Abelard (1079–1142), avoided undue EI by replacing the Conversions with one-way entailments. This solution, which weakens the Square, was never noticed in the history of logic.) Klima (1988) and Parsons (2008) introduce a zero element ø as a possible substitution: when a restrictor predicate extension $[\![F]\!] = Ø$, ø is an admissible substitution, producing falsity for all predicates, including F itself. Now, when $[\![F]\!] = Ø$, so that ø is the only admissible substitution, Some-x:F ¬[G(x)] (some F is not G) produces truth: G(ø) is false, hence ¬[G(ø)] is true, which satisfies the condition for Some-x:F. So it looks as if the Square has been saved. However, ø produces two paradoxes. (a) If ø produces falsity for all predicates, how about the predicate ‘be the zero element’? (b) Does or doesn’t ø belong to the extension of a predicate? If it does, why does it produce falsity? If it does not, why is it an admissible substitution? Moreover, this solution is intuitively rebarbative, as e.g. *Some centaurs are not married* counts as true, but *Some centaurs are bachelors* as false. We have here a logic that is faulted by an untenable ontology. The alternative is to treat EI as a presupposition induced by extensional term positions under predicates and to embed the unscathed bivalent Square in a trivalent presuppositional coating that protects it from presupposition failure. This way, the logical system is an organic part of the human ecology of language, mind and world and it helps explaining certain central facts of language and cognition.

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The Classical Aristotelian hexagon versus the Modern Duality hexagon

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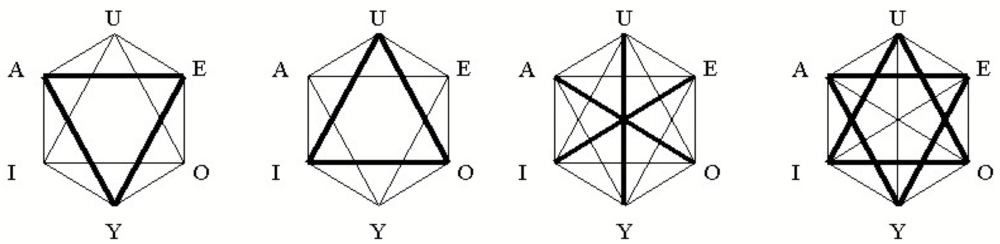
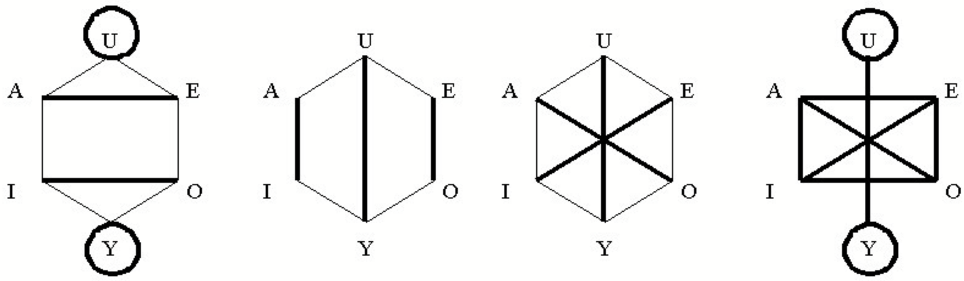
Peters & Westerståhl (2006, 2010) draw a crucial distinction between the “classical” Aristotelian squares of opposition and the “modern” Duality squares of opposition. The classical square involves four opposition relations, whereas the modern one only involves three of them: the two horizontal connections are fundamentally distinct in the Aristotelian case (contrariety vs subcontrariety) but express the same Duality relation of internal negation. Furthermore, the vertical relations in the classical square are unidirectional, whereas in the modern square they are bidirectional. The present paper argues that these differences become even bigger when two more operators are added, namely the U (= A □ E, all or no) and Y (= I □ O, some but not all) of Blanché (1969). In the resulting Aristotelian hexagon the two extra nodes are perfectly integrated, yielding two interlocking triangles of (sub)contrariety. In the duality hexagon by contrast, they do not enter into any relation with the original square, but constitute a independent pair of their own, since they are their own internal negations. Hence, they not only stand in a relation of external negation, but also in one of duality. This reflexive nature of the internal negation will be shown to result in defective monotonicity configurations for the pair, i.c. the absence of right-monotonicity (on the predicate argument). In a second part, we present an overview of those hexagonal structures which are both Aristotelian and Duality configurations, and those which are only Aristotelian.



contrary + *subcontrary* + *contradiction* = *Aristotelian Hexagon*



subnegation + dual + negation = Duality Hexagon

Many Valued Logics and Some Variations of the Square

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Besides the square of opposition there are some extended figures including more than the four corners. Extensions can be given with respect to the hierarchical structure of the square if one adds intermediately strong expressions between A und I, for example *most* between *all* and *some* or *probably* between *certainly* and *possibly* (in a probabilistic understanding). Another extension results from the combination of operators from different squares in one figure. In medieval logic such octagon was developed for modal predicate logic. But it is applicable for other combinations as well, for example space/time.

The aim of my talk is to show that both structures reoccur in the context of assertive one-place operators in many-valued logics, so called veridications. Added to a fuzzy logic or any logic where, semantically, the truth values represent one ordering they yield the hierarchical extension given above. But if veridication is added to a logic like four-valued FDE with another dimension besides truth, namely determination, we get a multi-level structure like in the medieval octagon

Squares for a Logical Calculus of Change

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We present a logical calculus of change, called LC, and illustrate some elementary relations between formulas by geometrical objects like squares and cubes. The language of LC is that one of classical sentential logic enriched by an operator C to be read as "it changes that …". A typical axiom is e.g. "CA implies Cnot-A", a basic rule is "From A you may infer not-CA" (theorems don't change). It can be proved that LC is complete in some semantics based on the notion of "history". The formula CA is true at some stage n of such a history iff its value differs from the stage n to n+1. If we consider formulas like CCA or not-CCA it turns out that the truth value of A changes following some rhythm. Different types of such rhythms can be illustrated by squares.

K. ŚWIĘTORZECKA, *Classical Conceptions of the Changeability of Situations and Things Represented in Formalized Languages*, The Publishing Company of the University of Cardinal St. Wyszyński in Warsaw, 2008, pp. 240.

Prohibition and Silence in the Logical Square

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This work consists in an investigation on the conditions for the existence of language, within the approaches of the Discourse analysis, as proposed by Pêcheux, and lacanian psychoanalysis. For such task we treat both interdiction and silence as constituents and founders of the discourse. We use the logic square with Aristotelian aletic modalities: The impossible, the possible, the necessary and the contingent. We also use Blanché's hexagon. However, our approach is not Aristotelian. It is based on discourse analysis and psychoanalytical reflections. Specifically, we use the lacanian principle which states the excluded (or the contradiction) as the founder the possible. We make here an exercise to see how the square would turn out using this principle. As part of this work's goals, we propose and create the square of saying or of utterances.

Neurolinguistic Topology and the Square of Opposition

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The Aristotelian Logical Square of Opposition (LSO) is governed by a fixed set of formal constraints which may be treated as independent variables without compromising the underlying configuration of a bivariate plot. Topological specification allows for comparative analysis with a variety of quaternary schemas, e.g., alethic generalizations of the LSO exchange lexical content while preserving the asymmetry of logical entailments; semiotic generalizations preserve alterity relations while suspending nominally existential requirements. The quaternary neurolinguistic asymmetry native to the primate cerebral cortex shares several topological and topographic features with the LSO. The conjunction of frontal-occipital asymmetry with specialized hemispheric lateralization is functionally expressed as the Phonological-Articulatory Loop (PAL) which governs the recursion of execution to planning for speech, communicative gesture and goal-directed hand actions. Improved brain imaging and single-neuron recordings now allow for the fine-grained isolation of syntactic, semantic and pragmatic functions at various scales. Formal problems traditionally associated with the LSO, such as the ambiguity of particular negation, are found to be homeomorphic to nonverbal causal attribution schemas. While traditionally perceived as a deficit, the defining ambiguities of the LSO are seen as crucial to cognitive scaffolding in general.

Paraconsistent Negation and the Square of Opposition

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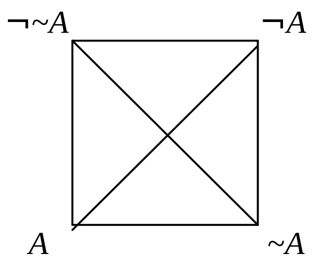
My aim is to show that the square of opposition is a handy tool for simple and accurate representation of properties of some non-classical negations. As an example I shall consider the paraconsistent negation of the logic CLuN, developed by Diderik Batens, which allows for gluts with respect to paraconsistent negation. CLuN is obtained by dropping from the classical semantics the following requirement for negation:
if A is true, then the negation of A is false and by keeping its converse: *if A is false, then the negation of A is true.*

Let ¬ stand for classical negation and ~ stand for paraconsistent CLuN negation. Semantical relations between a formula *A* and its classical and paraconsistent negations may be adequately depicted by the following version of the square:



In an analogous way may be described relations between a formula and its negations in the case of the logic CLaN (which allows for gaps with respect to negation), also developed by Batens.

A Mathematical Model for A/O Opposition in Scientific Inquiry.

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Of all of the relations on the square of opposition, contradiction is the most durable and apparently vital in any dialectical account of the power of logical relations. The force of refutation by contradiction is recognized as fundamental in theories of argument of all sorts. Nevertheless an examination of the dialogical use of contradictions in the history of science, presents a more nuanced picture. Successful processes of inquiry include long periods when contradictions are disregarded in the interest of continuing inquiry. This is readily seen in the history of, perhaps, the most successful inquiry project in human history, that is the development of a coherent and far-reaching image of the material world as exemplified buy the Periodic Table of Elements. Throughout the history of the introduction, elaboration and extension of the basic insight of periodicity, generalizations drawn from empirical fact and theory were confronted by large bodies of inconsistent evidence. This, contrary, to the dogma of falsification, did not result in the abandonment of such generalizations, rather the generalizations were used as the basis for additional experiments and theoretic elaborations, as often as not, leading to continuing empirical and theoretic success. If such inquiry procedures are to be seen as reasonable, a rather different account of contradiction than the standard must be attempted. The key to such an account of contradiction is to see truth as an emergent property across inquiry, rather than through a simple correspondence relation. In addition the locus of argument in inquiry needs to be moved from micro-arguments, seen as a recursive basis for more elaborate arguments, to a notion of arguments as embedded in fields, network of generalizations. Truth rather than being assigned to elements is seen as a property of elements in relation to their place in the field. Truth is a field property defined on emergent relations across a network, as in Quine's metaphor of the 'web of belief.'

In this paper I review the salient characteristics of a mathematical model for emerging truth (MET), and connect it with the notions of warrant and backing borrowed from Stephen Toulmin's classic model of argument. The MET affords an intuitive account of warrant strength, where warrants are seen as covering generalizations that support acceptable inference within a field. Inferences are non-classical in that they are nonmonotonic and open to approximations defined in terms of neighborhood relations in the field and subject to the constraints within a field. I take this as an elaboration of Toulmin's notion of backing. Counter-examples function dialectically within the context of the warrants that they contradict and as a function of their embeddings in countervailing systems of warrants. The paper concludes with principled criteria for evaluating the dialectical acceptance or rejection of counter-exams to generalizations (A/O Opposition)

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